Seat No. : _______________

AM-106

August-2021

B.Sc., Sem.-V

304 : Mathematics

(Mathematical Programming)

Time : 2 Hours] **[Max. Marks : 50**

Instructions : (1) Attempt any **THREE** questions from Q-**1** to Q-**8**.

(2) **Q-9** is **compulsory** question of short questions.

(3) Notations are usual, everywhere.

(4) Figures to right indicate marks of the question/sub-question.

1. (A) Prove that $E \subset R^n$ is convex set if and only if every finite convex linear combination of points in E also belongs to E. **7**

(B) A company makes two types of belts. Belt X is high quality belt and belt Y is of lower quality. The respective profits are ₹ 4 and ₹ 3 per belt. Each belt of type X requires twice as much time as a belt of type Y and if all belts were of type Y, the company could make 1000 per day. The supply of leather is sufficient for only 800 belts per day (both X and Y combined). Belt X requires a fancy buckle and only 400 per day are available. There are only 700 buckles a day available for belt Y. Formulate the LPP to determine the optimal product mix. **7**

2. (A) Prove that an intersection of two convex sets is a convex set. Is union of two convex sets convex always ? Justify your answer. **7**

(B) Mr. Khan required at least 10, 12 and 12 units of chemicals A, B and C for his garden. One liter of liquid product contains 3, 2 and 1 units of A, B and C respectively. A dry product contains 1, 2 and 4 units of each A, B and C per kilogram. If the liquid product sells for ₹ 3 per liter and the dry product sells for ₹ 2 per kilogram. Formulate the LPP to minimize the cost and meet the requirements. **7**

3. (A) Prove that the set of all feasible solution of an LPP is closed convex set which is bounded below. **7**

(B) Solve the following LPP by Simplex method : **7**

$$\text{Max } Z = 2x_1 + 4x_2 + 3x_3 + x_4$$

$$\text{s.t. } x_1 + 3x_2 + x_4 \le 4,$$

$$2x_1 + x_2 \le 3,$$

$$x_1 + 4x_3 + x_4 \le 3$$

$$x_j \ge 0, j = 1, 2, 3, 4.$$

4. (A) Solve the following LPP by big-M method. **7**

$$\text{Min } Z = 2x_1 + x_2 + 3x_3$$

$$\text{s.t. } x_1 + x_2 + 2x_3 \ge 5$$

$$2x_1 + 3x_2 + 4x_3 = 12$$

$$x_j \ge 0, j = 1, 2, 3.$$

(B) Solve the following LPP by two phase method. **7**

$$\text{Min } Z = -3x_1 + x_2 + x_3$$

$$\text{s.t. } x_1 - 2x_2 + x_3 \le 11,$$

$$-4x_1 + x_2 + 2x_3 \ge 3,$$

$$2x_1 - x_3 = -1$$

$$x_j \ge 0, \quad j = 1, 2, 3.$$

5. (A) State and prove the fundamental theorem of duality. **7**

(B) Solve the following LPP by Dual Simplex method : **7**

$$\text{Min } Z = 3x_1 + 5x_2$$

$$\text{s.t. } x_1 + x_2 \ge 1,$$

$$2x_1 + 3x_2 \ge 2 \text{ and } x_j \ge 0, j = 1, 2.$$

6. (A) Prove that the dual of dual is primal for LPP. **7**

(B) Use principle of duality to solve the following LPP : **7**

$$\text{Min } Z = 4x_1 + 3x_2 + 6x_3$$

$$\text{s.t. } x_1 + x_3 \geq 2,$$

$$x_2 + x_3 \geq 5 \text{ and } x_j \geq 0, j = 1, 2.$$

7. (A) Prove that transportation problem has a feasible solution if and only if it is balanced. **7**

(B) Solve the following assignment problem : **7**

	A	B	C	D
I	40	35	38	41
II	42	35	34	40
III	38	34	34	37

8. (A) Prove that an (m × n) balanced transportation problem has (m + n – 1) number of basic variables. **7**

(B) Solve the following transportation problem by MODI method : **7**

	D_1	D_2	D_3	a_i
O_1	16	20	12	200
O_2	14	8	18	160
O_3	26	24	16	90
b_i	180	120	150	450

9. Attempt any **FOUR** of the followings in short : **8**

(a) Determine whether the set $\{(x, y) \in R^2 \,|x^2 + y^2 \geq 3\}$ is convex or non-convex.

(b) Define Vertex. Also write the number of vertices of a tetrahedron.

(c) Define : A degenerate basic feasible solution and an optimum solution.

(d) When a LPP is said to have an unbounded solution and no feasible solution ?

(e) State any two differences between a Transportation problem and an Assignment problem.

(f) Find the initial basic feasible solution to the following T.P. by North-West Corner method :

	D_1	D_2	D_3	D_4	a_i
O_1	11	13	17	14	250
O_2	16	18	14	10	300
O_3	21	24	13	10	400
b_i	200	225	275	250	