Seat No. : _______________

TP-111

B.Sc. Sem.-III
May-2013

Core 201 Mathematics
(Advanced Calculus – I)

Time : 3 Hours] **[Max. Marks : 70**

Instruction : All questions are compulsory and carry equal marks.

1. Attempt any **two :**

(a) Define the limit of function f(x, y) and find the limit using definition :

$$\lim_{(x, y) \to (2, 1)} \frac{2x + y}{3y - x}$$

(b) Define the directional derivative of a function of several variables and find the directional derivative of $f(x, y) = \frac{xy^2}{x^2 + y^4}$, if $(x, y) \neq (0, 0)$ & $f(x, y) = 0$, if $(x, y) = (0, 0)$ at point (0, 0) along the direction of the vector (1, 1).

(c) Define the iterated limit of functions of two variables and find that limit for functions :

(i) $f(x, y) = \frac{\sin(x + y)}{x + y}$ at point (0, 0)

(ii) $f(x, y) = \left(\frac{x^2 + y^2}{x - y}\right)$ at point (1, 1)

2. Attempt any **two :**

(a) State and prove Young's theorem. Is converse true ? Justify.

(b) State and prove Schwartz's theorem. Is converse true ? Justify.

(c) Discuss the differentiability and continuity of function

$f(x, y) = \frac{xy^2}{x^3 + y^3}$, $(x, y) \neq (0, 0)$ & $f(x, y) = 2$, $(x, y) = (0, 0)$ at point (0, 0).

3. Attempt any **two** :

(a) State and prove Euler's theorem for a homogeneous function of two-variables with degree m.

(b) Find three positive integers whose sum is 15 and their product is maximum.

(c) If $z = \tan^{-1}\left(\frac{x^3 + y^3}{x + y}\right)$ then prove that $x^2 \frac{\partial^2 z}{\partial x^2} + 2xy \frac{\partial^2 z}{\partial x \partial y} + y^2 \frac{\partial^2 z}{\partial y^2} = \sin 4u - \sin 2u$

4. Attempt any **two :**

(a) Define : The radius of curvature of the curve. Derive the formula for the radius of curvature of the curve $y = f(x)$.

(b) Find the radius of curvature of following curves at origin :

(1) $y^2 = 4ax$

(2) $x^2 + 2xy + 2y^2 - 4x = 0$

(c) Find the double points of the curve $x^3 + 3x^2 - y^2 + 3x - 2y = 0$ and discuss their nature.

5. Give the answer of following questions :

(1) Define continuity of function of two variables.

(2) Define differentiation of function of two variables.

(3) State the Maclaurin's theorem for expansion of function of two variables.

(4) Find the radius of curvature of following curves : $x^2 + y^2 = 9$

(5) Define the curvature of the curve.

(6) Define : Double points, Cusp and Node

(7) State the necessary and sufficient condition for extreme values of functions of two variables.
