Seat No. : _______________

JK-110

January-2021

B.Sc., Sem.-V

EC-305 : Mathematics
(Discrete Mathematics)

Time : 2 Hours] **[Max. Marks : 50**

Instructions : (1) Attempt any **three** questions from Q-**1** to Q-**6**.
(2) Q-**7** is compulsory.
(3) Notations are usual, everywhere.
(4) Figures to the right indicate marks of the question/sub-question.

1. (A) State distributive Inequality and prove any one of them. **7**

(B) Explain Hass Diagram and also draw the Hass Diagram of (S_{2020}, D). **7**

2. (A) Define Lattice. **7**

Let n be a positive integer and S_n be the set of all positive factors of n. For every $a, b \in S_n \cdot aDb$ means "a divides b".

Then show that $\langle S_n, D\rangle$ is a lattice.

(B) For a Lattice $(L, \leq)$ prove that $a \leq b \Leftrightarrow a * b = a \Leftrightarrow a \oplus b = b, \forall a, b \in L$. **7**

3. (A) State De'Morgan's law and prove any one of them. **7**

(B) Define Direct product of two lattices and draw the Hass Diagram of $\langle S \times L, D\rangle$ where set S and L is divisors of 9 and 4 respectively. **7**

4. (A) Prove that every chain is Distributive Lattice. **7**

(B) Show that $(s_{30}, *, \oplus)$ and $(P(A), \cap, \cup)$ are isomorphic Lattice for $A = \{a, b, c\}$. **7**

5. (A) Show that there is no Boolean algebra of order 3. **7**

(B) Express $x_1 * x_2$ as Product of Sum(POS) canonical form in three variables. **7**

6. (A) State and prove stone representation theorem. **7**

(B) Define equivalent Boolean expression and also show that $(x \oplus y) * (x' \oplus z)$ and $(x * z) \oplus (x' * y)$ are equivalent. **7**

7. Attempt any **four** of the following in short : **8**

(a) Define: Partially ordered set.

(b) Give a relation on the set which is Transitive but neither reflexive nor symmetric.

(c) Find complement of each element in the set of divisors of 12.

(d) Define : Atom.

(e) Define : Minterm.

(f) Is the Boolean expression $\alpha(x, y, z) = (x * y * z) \oplus (x * y * z')$ symmetric ? Justify your answer.

Seat No. : _______________

JK-110

January-2021

B.Sc., Sem.-V

EC-305 : Mathematics
(Number Theory)

Time : 2 Hours] **[Max. Marks : 50**

Instructions : (1) Attempt any **three** questions from Q-**1** to Q-**6**.
(2) Q-**7** is compulsory.
(3) Notations are usual, everywhere.
(4) Figures to the right indicate marks of the question/sub-question.

1. (A) Show that the linear Diophantine equation $ax + by = c$ has solution if $d \mid c$, where $d = \gcd(a, b)$. Also if x_0 and y_0 are any solution of the equation then any other solution is given by $x = x_0 + \left(\frac{b}{d}\right)t$, $y = y_0 - \left(\frac{a}{d}\right)t$, where $t \in \mathbb{Z}$. **7**

   (B) Using Euclidean Algorithm, find the integers x, y, z satisfying

   $\gcd(198, 288, 512) = 198x + 288y + 512z$ **7**

2. (A) Prove : For given integers a & b with $b > 0$, there exist unique integers q & r such that $a = qb + r$, $0 \le r < b$. **7**

   (B) Obtain the general solution of the linear diophantine equation :

   $172x + 20y = 1000$ and hence solve for positive integers. **7**

3. (A) Show that every positive integer $n > 1$ can be expressed as a product of primes in unique way (does not matters of the orders of the prime factors). **7**

   (B) Prove : $p \ge q \ge 5$ & p, q primes $\Rightarrow 24 \mid p^2 - q^2$. **7**

4. (A) Prove that there is an infinite number of primes of the form $4k + 3$. **7**

(B) Using Chinese Remainder Theorem, solve : $17x \equiv 9 \pmod{276}$. **7**

5. (A) Show that the quadratic congruence $x^2 + 1 \equiv 0 \pmod{p}$, p = prime > 2 has solution if and only if $p \equiv 1 \pmod{4}$. **7**

(B) Prove : $7 \mid 5^{2n} + 3 \cdot 2^{5n-2}, \forall n \geq 1$. **7**

6. (A) Prove : If p is a prime then $(p - 1)! \equiv -1 \pmod{p}$.

Also by Wilson's theorem verify that $4(29!) + 5!$ is divisible by 31 or not. **7**

(B) State the Euler's Theorem and prove that $a^{1729} \equiv a \pmod{1729}$, $a \in Z$.

Is 1729 pseudo prime or absolute pseudo prime or both ? **7**

7. Attempt any **four** of the followings in short : **8**

(a) What is the unit digit of 3^{3^3} ?

(b) Find a prime P such that the numbers $p^2 + 8$ and $p^3 + 4$ are also primes.

(c) State the Well Ordering Principle and the fundamental theorem of arithmetic.

(d) Find the remainder when 2019^{2021} is divided by 2020 ?

(e) Find the missing digits X, Y, Z if 396 divides the number 3X6Y9Z.

(f) Obtain a set of any five numbers which is complete set of residues modulo 5.

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JK-110

January-2021

B.Sc., Sem.-V

EC-305 : Mathematics
(Financial Mathematics)

Time : 2 Hours] **[Max. Marks : 50**

Instructions : (1) Attempt any **three** questions from Q-**1** to Q-**6**.
(2) Q-**7** is compulsory.
(3) Notations are usual, everywhere.
(4) Figures to the right indicate marks of the question/Sub-question.

1. (A) What is the Present value of ₹ 21,00,000 received after ten years, for opportunity cost (interest rate) is 5% per year compounded annually, semi-annually, quarterly, monthly, weekly, daily and continuously ? **7**

   (B) Write a short note on Interest rates. **7**

2. (A) What is the Future value of ₹ 2,01,000 invested for 7 years, for opportunity cost (interest rate) is 7% per year compounded quarterly, weekly, daily, continuously ? Also find effective rate of interest in each case. **7**

   (B) Define bonds, shares, & index, also define arbitrage. **7**

3. (A) Define Bond, also define bond redemption at par, at premium and at discount. **7**

   (B) Consider a monthly coupon bond with annual coupon rate 12%, with the face value ₹ 1000 for 3 years tenure, consider the annual rate of interest in effect is 24%. Find the NPV for this bond. **7**

4. (A) Show that for a bond of n years with annual coupon payment C and face value F , if its yield (yield to maturity) is λ then its price is given by $P = \frac{1}{(1+\lambda)^n} \left[\frac{(1+\lambda)^n - 1}{\lambda} C + F\right]$. **7**

(B) Consider a portfolio with two bonds A and B, immunize this bonds portfolio using Macaulay duration, if the amount to be invested in portfolio P = 1,00,000 and for the duration of 3 years. Here Bond A is zero coupon bond of 2 years and Bond B is of 5 years annual coupon bond with annual coupon payment 300 and the face value of 1000 with the desired yield to maturity 7% continuously compounded. Then determine the amount of Money to be invested in each bond. **7**

5. (A) Write the Markowitz portfolio optimization problem with short selling and derive constrained optimization method using langrage's multiplier. **7**

(B) Calculate the portfolios mean return and variance using the following details : $R = (0.39, 1.16, -0.59)^T$, $W = (0.3, 0.2, 0.5)$ and

$$CV = \begin{bmatrix} 1.02 & 1.14 & 0.29 \\ 1.14 & 2.20 & 0.60 \\ 0.29 & 0.60 & 1.32 \end{bmatrix}$$ find $\bar{r}$ & σ^2 for portfolio. **7**

6. (A) Write a short note on portfolio diagram and choice of asset. **7**

(B) Consider a portfolio of three assets A, B & C with the following properties :

$\bar{r}_A = 0.4, \bar{r}_B = 0.3, \bar{r}_C = 0.7,$

$\sigma_A = \sigma_B = \sigma_C = 1$ & $\sigma_{AB} = \sigma_{BC} = \sigma_{AC} = 0$

For fixed $\bar{r} = 0.7$ find the minimum variance portfolio. **7**

7. Attempt any **four** of the followings in short : **8**

(a) Define an Asset.

(b) Define return and rate of return.

(c) Define Net present Value of given cash flow.

(d) Define Perpetuity.

(e) Define Efficient Frontier.

(f) Write the statement of one fund theorem.
