Seat No. : _______________

SM-107

September-2020

B.Sc., Sem.-VI

311 : Mathematics
(Convex Analysis and Probability Theory)

Time : 2 Hours] **[Max. Marks : 50**

Instructions : (1) Attempt any **THREE** questions in **Section – I**.

(2) **Section-II** is a **compulsory** section of short questions.

(3) Notations are usual everywhere.

(4) The right hand side figures indicate marks of the sub question.

Section – I

Attempt any **three** of the following questions :

1. (A) Define convex set and affine set. Also explain each of them by an example. **7**

(B) Show that the function $f : R \to R$ defined as $f(x) = x^2$ is monotonically increasing on $[0, \infty)$ and decreasing on $(-\infty, 0]$. **7**

2. (A) If the polynomial function $f : R \to R$ is defined as **7**

$$f(x) = x^4 - 8x^3 + 22x^2 - 24x + 7$$

then check the differentiability and monotonicity of f.

(B) Define convex and concave functions on an interval I. Also show that the function $f : R \to R$ defined as $f(x) = x^3$ is a convex function on $[0, \infty)$ whereas concave on $(-\infty, 0]$. **7**

3. (A) Define terms: Sample space, Impossible and Certain Events, Mutually exclusive and Exhaustive events, Difference events. Also, state the probability of certain events and complementary events. **7**

(B) A balanced dice is tossed twice. Write the elements of the following :

(1) Sample space.

(2) A = Event that sum of the integers on two dice is 7 or 10.

(3) B = Event that integers on dice are odd.

(4) C = Event that sum of integers on two dice is divisible by 3.

(5) D = Event that sum of integers on two dice is greater than 6.

Check whether events A and D are mutually exclusive or not. Also, find probabilities of events A, B, C and D. **7**

4. (A) Define Classical definition of probability.

State additive rule of probability for two and three events. If two events A, B and C are mutually exclusive events, then state the values of $P[A \cup B]$ and $P[A \cup B \cup C]$. Given two events A and B such that $P[A] = 0.32$, $P[B] = 0.50$ and $P[A \cup B] = 0.75$, then find $P[A \cap B]$. **7**

(B) What do you understand by an objective and subjective probability ? Give one example of each. Which kind of the probability can be explained mathematically ? Why ? **7**

5. (A) State the probability function of Binomial distribution.

State applications of binomial distribution and write its mean and variance. If a random variable X follows a binomial distribution with parameters $n = 6$, $p = 40$, then find mean, variance and moment generating function of X. Is mean larger than variance ? **7**

(B) If a random variable X follows poisson distribution with parameter $m = 2$, state values of mean (E(X)), variance (V(X)), $E(X + 4)$, $E(2X - 3)$, $V(4X)$. Also, find $P(X = 1)$, $P(X < 2)$. **7**

6. (A) If a random variable X follows binomial distribution with parameters (n, p) then, state the conditions to get poisson distribution from binomial distribution. State the mean and variance of Poisson distribution. **7**

If, a random variable X follows Poisson distribution with parameter θ, then, find θ such that $P[X = 2] = P[X = 3]$. Also, if $\theta = 2$, find $P[X = 0]$, $P[X = 1]$

$\{e^{-1} = 0.368, e^{-2} = 0.135, e^{-3} = 0.050\}$

(B) State the probability function of a normal distribution. Also, state the relationship between mean, median and mode of a normal distribution. Do you agree that the normal distribution is symmetric one ? **7**

Section – II

7. Answer any **Four** of the followings in Short : **8**

(1) Give examples each one of convex and non-convex sets of R^2.

(2) If $A = \{(x, y) \in R^2 / x^2 + y^2 \leq 5\}$, then find the convex hull of A.

(3) Using the addition rule of probability for two events A and B defined on a finite sample space such that $P[A] = 0.35$, $P[B] = 0.45$ and $P[A \cap B] = 0.65$, then find the probability of events $P(A \cup B)$ and $P(\overline{A} \cap \overline{B})$.

(4) State Bayes' theorem on probability. Also, state its uses.

(5) Give one application, each of binomial and poisson distributions.

(6) State moment generating function of binomial distribution.

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SM-107

September-2020

B.Sc., Sem.-VI

311 : Mathematics
(Cryptography)

Time : 2 Hours] **[Max. Marks : 50**

Instructions : (1) Attempt any **THREE** questions in **Section – I**.

(2) **Section-II** is a **compulsory** section of short questions.

(3) Notations are usual everywhere.

(4) The right hand side figures indicate marks of the sub question.

Section – I

Attempt any **three** of the following questions :

1. (A) Define Ring. Explain Euclidean algorithm. **7**

(B) Obtain the value of x that satisfies the following four congruence $x \equiv 1(\text{mod}2)$, $x \equiv 2(\text{mod}3)$, $x \equiv 3(\text{mod}5)$, $x \equiv 4(\text{mod}7)$. **7**

2. (A) If n is a fixed positive integer and a, b, c, d are integer, then prove that the following : **7**

(1) $a \equiv b \pmod{n} \Leftrightarrow b \equiv a \pmod{n} \Leftrightarrow a - b \equiv 0 \pmod{n}$.

(2) $a \equiv a \pmod{n}$.

(3) $a \equiv b \pmod{n}$ and $b \equiv c \pmod{n} \Leftrightarrow a \equiv c. \pmod{n}$.

(B) Obtain all the primite element of $\mathbb{Z}_{37}$. **7**

3. (A) Define Cryptosystem. Encrypt the following message using a shift cipher with a shift of + 20. "Comfort is the enemy of achievement." **7**

Encrypt the following message using a shift cipher with a shift of – 20. "The person that you will spend the most time with in your life is yourself, so you better try to make yourself as interesting as possible."

(B) Using Affine cipher encrypt "People are not against you; they are for themselves." with $(4x + 5) \pmod{26}$. **7**

4. (A) A ciphertext obtained using the shift cipher is given below. Do the cryptanalysis and obtain the plaintext.: HAAHJRHAKHDU. **7**

(B) Suppose that affine cipher $E(x) = (ax + b)(\text{mod } 26)$ enciphers s as U and o as A. Find a and b. **7**

5. (A) Define Trapdoor function. Discuss Birthday Paradox. **7**

(B) Alice and Bob select the prime number p = 17 with g = 6 as a primitive elements. Alice select a random number a = 5 as private key, computes her public key and sends it to Bob; Bob uses b = 9 as the ephemeral key to mail a message m = 13 to Alice. Show the full transaction including the recovery of massage key using ElGamal Public-Key cryptosystem. **7**

6. (A) Alice selects p = 23 and c = 5 and convey the same to Bob Alice selects a = 6 and Bob selects b = 15. What is private key exchange between them using the DH algorithm ? Show how Eve mounts an attack using Shank's algorithm and wrenches the private key shared between Alice and Bob. **7**

(B) With p = 17, q = 19, e = 29 and m = 25. Show that the complete transaction conforming to the RSA cryptosystem. **7**

Section – II

7. Attempt any **Four**. Do as directed : **8**

(1) a is not primitive element of $\mathbb{Z}_p$ if ________.

(2) $\phi(n_1, n_2) = \phi(n_1)\ \phi\ (n_2)$ if ________.

(3) Explain the terms in the context of cryptography: Encryption, Diagram, Trigram.

(4) ________ and ________ are specific case of Polycryptosystem.

(5) A combination of an encryption algorithm and a decryption algorithm is called a ________.

(6) Full form of RSA.

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SM-107

September-2020

B.Sc., Sem.-VI

311 : Mathematics
(Operations Research)

Time : 2 Hours] **[Max. Marks : 50**

Instructions : (1) Attempt any **THREE** questions in **Section – I**.
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Section – I

1. (A) Discuss Economic Order Quantity (EOQ) Model with finite replenishment rate. **7**

(B) Using the EOQ model with constant rate of demand, obtain EOQ and the total variable cost associated with policy of ordering quantities of that size. Annual demand = 10,000 units, ordering cost = ₹ 40 per order and inventory carrying cost is 20% of average inventory value. **7**

2. (A) Discuss Economic Order Quantity (EOQ) model with constant rate of demand. **7**

(B) A company plans to consume 760 pieces of a particular component. Pat records indicates that the purchasing department spent ₹ 12,555 for placing 15,500 Purchase orders. The average inventory was valued at ₹ 45,000 and the total Storage cost was ₹ 7650 which included wages, taxes, rent, insurance etc. related to the store department. The company borrows capital at the rate of 10% per year. If the price of component is ₹ 12 and the lot-size is 10, find the following :

(1) Purchase price per year (2) Purchase expenses per year (3) Storage expenses per year (4) Capital cost per year (5) Total cost per year. **7**

3. (A) Compare and contrast CPM and PERT. Under what conditions would you recommend scheduling by PERT? Justify your answer with reasons. **7**

(B) Draw an arrow diagram showing the following relationships. **7**

Activity	A	B	C	D	E	F	G	H	I	J	K	L	M	N
Immediate Predecessor	–	–	–	A, B	B, C	A, B	C	D, E, F	D	G	G	H, J	K	I, L

4. (A) Discuss various steps involved in the applications of PERT and CPM. **7**

(B) An established company has decided to add a new product to its line. It will buy the product from a manufacturing concern, package it, and sell it to a number of distributors selected on a geographical basis. Market research has indicated the volume expected and the size of sales force required. The steps shown in the following table are to be planned : **7**

Activity	**Description**	**Time (Weeks)**
A	Organize sales office	6
B	Hire salesmen	4
C	Train salesmen	7
D	Select advertising agency	2
E	Plan advertising campaign	4
F	Conduct advertising campaign	10
G	Design package	2
H	Setup packaging facilities	10
I	Package initial stocks	6
J	Order stock from manufacturer	13
K	Select distributors	9
L	Sell to distributors	3
M	Ship stocks	5

5. (A) Explain the two person zero sum game giving a suitable example. **7**

(B) Let the payoff matrix is as follow : $\begin{bmatrix} 40 & -80 \\ 15 & -20 \\ 20 & 50 \end{bmatrix}$. Determine optimal strategies and value of the game. **7**

6. (A) Explain Dominance Principle in Game theory. **7**

(B) Solve the following game whose payoff matrix is given by :

$$\begin{bmatrix} 3 & -1 & 4 & 6 & 7 \\ -1 & 8 & 2 & 4 & 12 \\ 16 & 8 & 6 & 14 & 12 \\ 1 & 11 & -4 & 2 & 1 \end{bmatrix}$$ **7**

Section – II

7. Attempt any **Four** short questions : **8**

(1) Give types of Direct inventory.

(2) Explain any two cost involved in Inventory problem.

(3) Explain Looping and Dangling.

(4) Explain Dummy activity.

(5) Give list of applications of PERT and CPM techniques in Project management.

(6) Explain Maximin and Minimax Principle in short.