3/22

1108E136

Candidate's Seat No :_______

M.Sc. Sem-3 Examination

502

Mathematics (Algebra-I)

August 2021

Time : 2-00 Hours]

[Max. Marks : 50

Instructions:

1. All the questions in **Section-I** carry equal marks.
2. Attempt any **Three** questions from **Section-I**
3. Questions in **Section-II** are **COMPULSORY**

Section-I

1. (A) Prove that, for $n \geq 3$, S_n is a non-Abelian group. [7]

   (B) Define group. Give with details an example of a group of order 45. . [7]

2. (A) Give with details an example of an infinite non-Abelian group. [7]

   (B) State (without proof) the Fundamental theorem of finite Abelian groups. How many Abelian groups of order 100 are there? Justify. [7]

3. (A) What is the order of the permutation $\alpha = (12)(2345)$ in the group A_{10}? [7]

   (B) Define simple groups. Determine the values of n for which the group $\mathbb{Z}_n$ is simple. [7]

4. (A) Define an automorphism. Prove that $Aut(\mathbb{Z}_n)$ is isomorphic to $U(n)$. [7]

   (B) Define a normal subgroup. Give an example of group G and its subgroup H such that H that is not normal in G. [7]

5. (A) Prove or disprove: The group $(\mathbb{R}, +)$ is isomorphic to the group $(\mathbb{Q}, +)$. [7]

   (B) Let $|G| = 100$ and H be a subgroup of G. write down all the possible orders of H? [7]

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6. (A) Define homomorphism. If $\phi : G \to G'$ is a homomorphism then prove that $\phi(e) = e'$. [7]

   (B) Define simple groups. If $|G| = p$ (where, p is a prime), show that G is simple. [7]

7. (A) What is the order of the group $U(15)$?Explain. [7]

   (B) Define the conjugacy class **cl(a)** of the element $a \in G$.

   When does $cl(a) = \{a\}$ hold for all $a \in G$? Explain. [7]

8. (A) How many homomorphism are there from the group $\mathbb{Z}_{12}$ to the group $\mathbb{Z}_{21}$? [7]

   (B) How many elements of order 2 are there in the group $\mathbb{Z}_{100} \oplus \mathbb{Z}_{200}$? [7]

Section -II

[8]

1. Any infinite cyclic group G is isomorphic to the group ____

   (A) $(\mathbb{R}, +)$
   (B) $(\mathbb{Z}, +)$
   (C) $(\mathbb{Q}, +)$
   (D) $(\mathbb{Z}_n, +_n)$

2. Let a and b be any two elements of group G. Then $|ab| = |ba|$ if and only if G is ____

   (A) Abelian
   (B) cyclic
   (C) finite
   (D) none of these

3. What is the order of the group $G = \mathbb{Z}_5 \oplus U(7)$?

   (A) 30
   (B) 32
   (C) 35
   (D) 34

4. How many elements of the group A_5 have order 3?

(A) 1

(B) 20

(C) 15

(D) 24

5. What is the order of the center $Z(S_3)$ of the group S_3?

(A) 6

(B) 4

(C) 1

(D) 2

6. Which of the following groups is simple?

(A) S_3

(B) $(\mathbb{Z}, +)$

(C) $(\mathbb{Z}_4, +_4)$

(D) $(\mathbb{Z}_7, +_7)$

7. For which values of m and n, the group $G = \mathbb{Z}_n \oplus \mathbb{Z}_m$ is cyclic?

(A) $m = 10$, $n = 15$

(B) $m = 12$, $n = 21$

(C) $m = 4$, $n = 28$

(D) $m = 10$, $n = 33$

8. What is the order of the group $Aut(\mathbb{Z}_{10})$?

(A) 10

(B) 1

(C) 4

(D) 2

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