Seat No. : ______________

AJ-103

August-2021

B.Sc., Sem.-V

301 : Mathematics

(Linear Algebra – II)

Time : 2 Hours] **[Max. Marks : 50**

Instructions : (1) Attempt any **three** questions from questions **1** to **8**.
(2) Question **9** is compulsory question.
(3) Notations are usual everywhere.
(4) The figure to the right indicate marks of the question/sub-question.

1. (A) If W is a subspace of a finite dimensional vector space V. Then prove that
$\dim W + \dim W^0 = \dim V$, where W^0 is annihilator of W. **7**

(B) Which of the following functions defined for vectors $u = (x_1, x_2)$ and $v = (y_1, y_2)$ in R^2 is a bilinear form ? **7**

(1) $f(u, v) = x_1y_2 - x_2y_1$

(2) $g(u, v) = (x_1 - y_1)^2 + x_2y_2$

2. (A) State and prove the dual basis existence theorem. **7**

(B) Find the dual basis of the basis B = {(1, –1, 3), (0, 1, –1), (0, 3, –2)} of the vector space V_3. **7**

3. (A) State and prove the Cauchy-Schwartz's inequality. **7**

(B) If <, > a function defined by $\langle x, y\rangle = x_1y_1 - x_2y_1 - x_1y_2 + 4x_2y_2$ for $x = (x_1, x_2)$ and $y = (y_1, y_2)$ in R^2 then, determine whether <, > is an inner product on R^2 or not. **7**

4. (A) Prove that any orthogonal set of non-zero vectors in an inner product space V is linearly independent. **7**

(B) Apply Gram-Schmidt orthogonalization process to the basis **7**
B = {(1, 1, 0), (1, 0, 0), (1, 1, 1)} in order to get the orthonormal basis for R^3.

5. (A) State and prove that Laplace Expansion. **7**

(B) Compute the detA if $A = \begin{bmatrix} 1 & 5 & 0 & 0 \\ 2 & 0 & 8 & 0 \\ 3 & 6 & 9 & 0 \\ 4 & 7 & 10 & 1 \end{bmatrix}$. **7**

6. (A) Suppose det : $V_n \longrightarrow R$ is a function satisfying properties of the determinant. Then prove that **7**

(i) $\det(v_1, v_2, \ldots v_i, \ldots v_j, \ldots v_n) = \det(v_1, v_2, \ldots v_i + \alpha v_j, \ldots v_j, \ldots v_n)$ whenever $i \neq j$.

(ii) $\det(v_1, v_2, \ldots v_i, \ldots v_j, \ldots v_n) = 0$ if $\{v_1, v_2, \ldots v_j, \ldots v_i, \ldots v_n\}$ is linearly dependent.

(B) Using Cramer's Rule (if applicable), solve the system of equations : **7**

$x + 2y + 3z = 3,\ 2x - z = 4,\ 4x + 2y + 2z = 5$

7. (A) State and prove the Cayley-Hamilton's theorem. **7**

(B) Diagonalize the quadratic equation $7x^2 + 7y^2 - 2z^2 + 20yz - 20zx - 2xy = 36$. **7**

8. (A) If $T : V \longrightarrow V$ is a symmetric linear map and if v_i are given vectors of T corresponding to eigen values λ_i, $i = 1, 2$ with $\lambda_1 \neq \lambda_2$ then prove that v_1 and v_2 are orthogonal vectors. **7**

(B) Apply the Cayley-Hamilton's theorem to find A^{-1} if $A = \begin{pmatrix} 3 & 1 & -1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{pmatrix}$. **7**

9. Answer any **four** of the following in short : **8**

(a) Let $T : V_3 \longrightarrow V_2$ and $S : V_2 \longrightarrow V_2$ be two linear maps define by $T(x_1, x_2, x_3) = (x_1 + x_2 + x_3, x_1)$ and $S(x_1, x_2) = (x_2, x_1)$, then determine ST.

(b) Let u = (1, 2, 3) and v = (–2, 3, 0), then find the vector projection of u on v.

(c) If $x = y$ and x and y are column vector of vector space V_2, then find the value of $\det(x, y)$.

(d) Find the real number $\propto$ such that the vectors $u = (2, \propto, 1)$ and $v = (4, -2, -2)$ are orthogonal.

(e) Find the matrix A if $A^{-1} = \begin{pmatrix} 1 & 3 \\ -2 & 6 \end{pmatrix}$.

(f) Find detA without expansion of the determinant if $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ 1 & 2 & 3 \end{bmatrix}$.