Seat No. : _______________

AE-114

August-2021

B.Sc., Sem.-VI

310 : Mathematics

(Graph Theory)

Time : 2 Hours] **[Max. Marks : 50**

Instructions : (1) Attempt any **THREE** questions in Section – I.

(2) Section – II is a compulsory section of short questions.

(3) Notations are usual everywhere.

(4) The right hand side figures indicate marks of the sub question.

SECTION – I

Attempt any **THREE** of the following questions :

1. (a) If G is any graph with e edges and n vertices $v_1, v_2, v_3, ... v_n$ then prove that

$$\sum_{i=1}^{n} d(v_i)\, 2e.$$ **7**

(b) Let G be a nonempty graph with atleast two vertices. If G is bipartite then prove that it has no odd cycles. **7**

2. (a) Define isomorphism of graphs. Show that the following graphs (Fig-l) are isomorphic. **7**

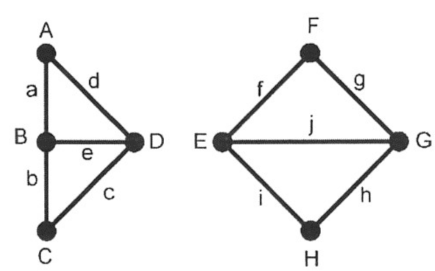

Fig. – 1

(b) Prove that the complete graph K_n has $\frac{n(n-1)}{2}$ edges. **7**

3. (a) Write down the adjacency and incidence matrices of the following graph (Fig-2). **7**

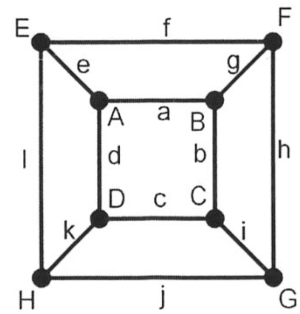

Fig. – 2

(b) Let G be a graph with n vertices. If G is connected graph with n – 1 edges then prove that G is a tree. **7**

4. (a) Let G be a graph with n vertices v_1, v_2, ..., v_n and let A denote the adjacency matrix of G w.r.t. this listing of the vertices. Let k be any positive integer and letA^k denote the matrix multiplication of k copies of A. Then prove that (i, j)th entry of A^k is the number of different $v_i – v_j$ walks in G of length k. **7**

(b) Prove that a connected graph G is a tree if and only if every edge of G is a bridge. **7**

5. (a) Give a list of all spanning trees, including isornorphic ones, of the complete graph K_4. **7**

(b) For the following graph (Fig-3), find (i) all cut vertices (ii) all bridges (iii) a spanning tree (draw it) (iv) connectivity and (v) all n for which it is n-connected. **7**

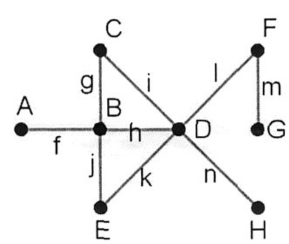

Fig. – 3

6. (a) Prove that if a vertex v of a connected graph G is a cut vertex of G then, there are two vertices u and w of G different from v such that v is on every u – w path in G. **7**

(b) Use Back-tracking algorithm to find a shortest path from a vertex A to a vertex M in graph (Fig-4). **7**

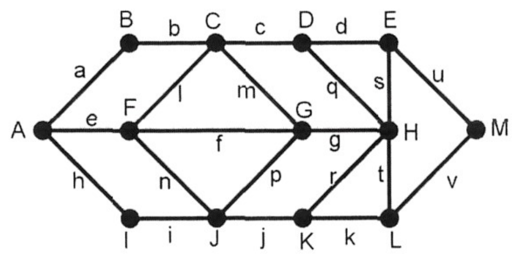

Fig. – 4

7. (a) If G is a graph in which the degree of every vertex is atleast two then prove that G contains a cycle. **7**

(b) Find closure of the graph (Fig-5) : **7**

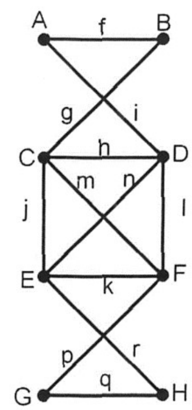

Fig. – 5

8. (a) Write a short note on Konigsberg seven bridges problem. **7**

(b) Use the Fleury's algorithm to produce an Euler tour for the following graph (Fig-6) **7**

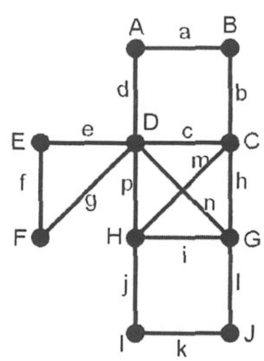

Fig. – 6

SECTION – II

9. Attempt any **FOUR** of the followings in short : **8**

(i) Define any two : (a) Loop (b) Parallel edges (c) Empty Graph.

(ii) Define k-regular graph and give an example.

(iii) Draw fusion graph from the graph in (Fig-6) by fusing vertices E and F.

(iv) Define : (a) Forest and (b) Bridge.

(v) A graph is disconnected. What is its connectivity ? Define spanning tree.

(vi) If connected graph G has 201 edges, what is the maximum possible number of vertices in G ? Why ?

(vii) Define : Hamiltonian Cycle. Is the graph in (Fig-5) Harniltonian ?

(viii) Define : (a) Euler trail and (b) Euler tour.
