2510E427

Candidate's Seat No :_______

B.Sc. Sem-6 Examination

CC 308

Mathematics

October 2021

Time : 2-00 Hours] [Max. Marks : 50

Instruction: (1) All Questions in Section I carry equal marks

(II) Attempt any THREE questions in Section I

(III) Question 9 in Section II is COMPULSORY

SECTION I

Q1 (A) Define Riemann integrable function. If $f \in R[a,b]$ then prove that $f^2 \in R[a,b]$. 7

(B) Let $f(x) = 3x$ on $[0,1]$. For $n \in \mathbb{N}$, define $P_n = \left\{0, \frac{1}{n}, \frac{2}{n}, \frac{3}{n}, \dots, \frac{n-1}{n}, 1\right\}$ then compute $\lim\limits_{n\to\infty} U_{P_n}$ and $\lim\limits_{n\to\infty} L_{P_n}$. Is the function integrable? If so, find the value of the integral. 7

Q2 (A) State and prove Second Fundamental Theorem of Calculus. 7

(B) State Second Mean Value Theorem of Integral Calculus. Find a point c in $[0, \frac{\pi}{2}]$ such that $\int_0^1 \frac{1}{1+x^2}\,dx = 1$. 7

Q3 (A) Let $\{x_n\}$ and $\{y_n\}$ be real sequences. Then prove that 7

(I) $\inf x_n \le \underline{\lim}\, x_n \le \overline{\lim}\, x_n \le \sup x_n$

(II) $\overline{\lim}\,(x_n + y_n) \le \overline{\lim}\, x_n + \overline{\lim}\, y_n$

(B) State comparison test. Hence show that series $\sum_{n=0}^{\infty} \frac{1}{n!}$ converges to the value e. 7

Q4 (A) Prove that every Cauchy sequence in $\mathbb{C}$ is bounded. Also prove that $\mathbb{C}$ is complete. 7

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(B) State condensation test. Hence check the convergence of $\sum_{n=2}^{\infty} \frac{1}{n(\log n)^{\alpha}}$, $\alpha \in \mathbb{R}$. 7

Q5 (A) State and prove Leibniz alternating series test. 7

(B) Define Conditionally convergent series. Check whether $\sum_{n=0}^{\infty}(-1)^n \frac{1}{\sqrt{n+1}}$ is conditionally convergent? 7

Q6 (A) If $\sum a_n$ converges absolutely, then prove that the series $\sum a_n$ converges. Is the converse true? Justify. 7

(B) Find the radius of convergence of the following power series whose n^{th} terms are given below: 7

1. $(n+1)z^n$
2. $\frac{n^3}{n!} z^n$

Q7 (A) State and prove Binomial series theorem. 7

(B) Derive Taylor's formula with the integral form of the remainder for $f(x) = \sin x$ about $a = 0$ in $(-\infty, \infty)$. 7

Q8 (A) Derive Taylor's formula with Cauchy form of the remainder for $f(x) = (1-x)^{1/2}$ about $a = 0$ and $-1 < x < 1$. 7

(B) Find a power series solution of $y'' + xy = 0$ with $y(0) = 1$ and $y'(0) = 0$. 7

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SECTION II

Q9 **Attempt any FOUR short questions:** **8**

(i) Give one function which is not Riemann integrable.

(ii) Discuss convergence of the series $\sum_{n=1}^{\infty} \frac{1}{2^n}$.

(iii) Give example of absolutely convergent series.

(iv) The Taylor's series of f converges to $f(x)$ at $x = a$ iff $R_n(x) \to$ __________ as $n \to$ __________. (Fill in the blanks)

(v) Is $\lim\limits_{n \to \infty} x_n = 0$ a sufficient condition for convergence of $\sum_{n=0}^{\infty} x_n$. Justify.

(vi) Does rearrangement of the terms in an infinite series may affect the sum? Justify by giving example.