Seat No. : _______________

# NE-123

**November -2021**

**B.Sc., Sem.-V**

**304 : Mathematics**

**(Mathematical Programming)**

**Time : 2 Hours]** **[Max. Marks : 50**

**Instructions :** (1) Attempt any **THREE** questions in Section – I.

(2) Section – II is a **compulsory** section of short questions.

(3) Notations are usual everywhere.

(4) The right hand side figures indicate marks of the sub question.

## SECTION – I

Attempt any **THREE** of the following questions :

1. (a) Define convex set. Show that the set $S = \{u \in R^n : \| u \| \le 1\}$ is convex. **7**

   (b) Prove or disprove : $S \subset R^n$ is convex if and only if the convex hull of S is S itself. **7**

2. (a) Let $S_1 = \{(x, y) \in R^2 : x^2 + y^2 \le 9\}$ and $S_2 = \{(x, y) \in R^2 : x^2 + y^2 \ge 4\}$. Obtain $S_1 \cap S_2$. Is $S_1 \cap S_2$ convex ? Justify your answer. **7**

   (b) An oil company owns refineries A and B. Refinery A can produce 20 barrels of petrol and 25 barrels of diesel per day. Refinery B can produce 40 barrels of petrol and 20 barrels of diesel per day. The company requires at least 1000 barrels of petrol and at least 800 barrels of diesel. If it costs 300 rupees per day to operate refinery A and 500 rupees per day to operate refinery B. How many days should each refinery be operated by the company to minimize cost? Formulate the problem as an LPP. **7**

3. (a) Solve the following LPP by simplex method, max $z = 2x_1 + 3x_2$ subject to $-x_1 + 2x_2 \le 4$, $x_1 + x_2 \le 6$, $x_1 + 3x_2 \le 9$. where $x_1, x_2$ are unrestricted in sign. **7**

   (b) Solve the following LPP by big M method.

   max $z = 3x_1 - x_2$, subject to $2x_1 + x_2 \ge 2$, $x_1 + 3x_2 \le 3$, $x_2 \le 4$, where $x_1, x_2 \ge 0$. **7**

4. (a) Find all the basic solutions of the following LPP without using simplex algorithm. Choose that one which maximizes z. **7**

max z = $2x_1 + 3x_2 + 4x_3 + 7x_4$, subject to $x_1 + 3x_2 - x_3 + 4x_4 = 8$,

$x_1 - 2x_2 + 6x_3 - 7x_4 = -3$, where $x_i \geq 0$, i = 1, 2, 3, 4.

(b) Solve the following LPP by two phase method. **7**

max z = $3x_1 - x_2$, subject to $2x_1 + x_2 \geq 2$, $x_1 + 3x_2 \leq 3$, where $x_1, x_2 \geq 0$.

5. (a) For an LPP prove that dual of dual is primal. **7**

(b) When dual simplex method is applicable to solve an LPP ? Use dual simplex method to solve the following LPP. **7**

$$\min z = 3x_1 + x_2$$

Subject to

$$x_1 + x_2 \geq 1$$

$$2x_1 + 3x_2 \geq 2$$

where $x_1, x_2 \geq 0$.

6. (a) Find the optimum integer solution to the following LPP. **7**

$$\max z = x_1 + 4x_2$$

Subject to

$$2x_1 + 4x_2 \leq 7$$

$$5x_1 + 3x_2 \leq 15$$

Where $x_1, x_2 \geq 0$ and are integers.

(b) Obtain dual of the following primal problem. **7**

$$\min z = x_1 - 3x_2 - 2x_3$$

Subject to

$$3x_1 - x_2 + 2x_3 \leq 7$$

$$2x_1 - 4x_2 \geq 12$$

$$-4x_1 + 3x_2 + 8x_3 = 10$$

Where $x_1, x_2 \geq 0$ and $x_3$ is unrestricted in sign.

7. (a) Prove that the number of basic variables in a transportation problem with $m$ origins and $n$ destinations is at most $m + n - 1$. **7**

(b) Obtain the initial basic feasible solution of the following transportation problem using Vogel's approximation method. Is the solution optimum ? Justify your answer. **7**

| | $D_1$ | $D_2$ | $D_3$ | $D_4$ | **Supply** |
|---|---|---|---|---|---|
| $O_1$ | 20 | 22 | 17 | 4 | 120 |
| $O_2$ | 24 | 37 | 9 | 7 | 70 |
| $O_3$ | 32 | 37 | 20 | 15 | 50 |
| **Demand** | 60 | 40 | 30 | 110 | |

8. (a) Explain degeneracy in transportation problem. Resolve degeneracy in the following transportation problem and obtain its initial basic feasible solution. **7**

| | $D_1$ | $D_2$ | $D_3$ | $D_4$ | **Supply** |
|---|---|---|---|---|---|
| $O_1$ | 2 | 2 | 2 | 4 | 1000 |
| $O_2$ | 4 | 6 | 4 | 3 | 700 |
| $O_3$ | 2 | 2 | 1 | 0 | 900 |
| **Demand** | 900 | 800 | 500 | 400 | |

(b) Find solution of unbalanced assignment problem using Hungarian method. **7**

| | **I** | **II** | **III** | **IV** |
|---|---|---|---|---|
| **A** | 9 | 14 | 19 | 15 |
| **B** | 7 | 17 | 20 | 19 |
| **C** | 9 | 18 | 21 | 18 |
| **D** | 10 | 12 | 18 | 19 |
| **E** | 10 | 15 | 21 | 16 |

## SECTION – II

9. Answer any **four** in short : **8**

(1) Write down convex hull of the set {(1 ,0), (0, 3)}.

(2) State fundamental theorem of duality.

(3) If there are *n* workers and *n* jobs, how many assignments can be given to the assignment problem ?

(4) State a general form of the transportation problem and state a necessary and sufficient condition for the existence of a feasible solution of the transportation problem.

(5) Define degenerate and non-degenerate basic feasible solution of an LPP.

(6) Explain by an example that union of two convex sets may not be convex.

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