

Seat No. : \_\_\_\_\_

**ND-139**

**November-2021**

**B.Sc., Sem.-V**

**303 : Mathematics  
(Complex Variables & Fourier Series)**

**Time : 2 Hours]**

**[Max. Marks : 50**

- Instructions :** (1) All Questions in SECTION – I carry equal marks.  
(2) Attempt any **THREE** questions in SECTION – I.  
(3) Question – 9 in SECTION – II is **COMPULSORY**

**SECTION – I**

1. (A) State and prove De-Moivre's theorem. 7  
(B) Find the cube root of  $-27i$  7
  
2. (A) Find all values of  $(-1 + i)^{2/5}$ . 7  
(B) Find the radius of convergence of the series  
(i)  $\sum \frac{z^n}{n^n}$  (ii)  $\sum \frac{n!z^n}{n^n}$  7
  
3. (A) State and prove sufficient condition for Cauchy-Riemann condition in Cartesian coordinates. 7  
(B) If  $u = x^3 - 3x^2y$  then prove that  $u$  is a Harmonic function and find the harmonic conjugate of  $u$ , also find analytic function  $f(z)$ . 7
  
4. (A) Show that  $u(x, y) = e^{-x} \cos y$  is harmonic function then find its harmonic conjugate and function  $f(x, y) = u + iv$ . 7  
(B) Define limit of a complex function  $f(z)$ . Prove that if the limit exists then it is unique. 7
  
5. (A) For  $w = Ze^{i\frac{\pi}{4}}\sqrt{2}$ , determine the region  $R$  of  $w$  – plane corresponding to the Rectangular region bounded by the lines 7  
 $x = 0, y = 0, x = 2, y = 3$  in  $Z$  plane.  
(B) State and Prove the necessary and the sufficient condition for the transformation to be Conformal. 7

6. (A) Under the transformation  $w = 1/z$  find the image of  $|z-2i| = 2$ . Sketch the region in both the plane. 7
- (B) Find the image of the curve  $|Z + i| = 2$  under the mapping  $W = \frac{Z+i}{Z-i}$ . 7
7. (A) If the series  $\frac{1}{2}a_0 + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$  converges uniformly to  $f$  on  $[-\pi, \pi]$ . Then prove that it is a Fourier Series for  $f$  on  $[-\pi, \pi]$ . 7
- (B) Find the Fourier Series to represent  $e^{ax}$  in the interval  $[-\pi, \pi]$ . 7
8. (A) If  $f(x)$  is Reimann Integrable in  $(-\pi, \pi)$  then the series  $\sum_{n=1}^{\infty} (a_n^2 + b_n^2)$  converges Where  $a_n, b_n$  are the Fourier Coefficients of  $f(x)$ . 7
- (B) Find the Fourier Series to represent  $f(x) = x + x^2$  in the interval  $[-\pi, \pi]$ . 7

### SECTION – II

9. Attempt any **FOUR** : 8
- (i) Express  $-\sqrt{3} + i$  in the polar form.
- (ii) Find the value of  $(1-i)^{14}$ .
- (iii) Is  $w = e^z$  entire ? Justify.
- (iv) Examine the continuity of the function  $\frac{Z}{Z-1}$
- (v) Find the point at which the mapping  $W = Z^2 + \frac{1}{Z^2}$  is not conformal.
- (vi) Find  $\int_{-\pi}^{\pi} \sin mx \cos nx dx$ .
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