Seat No. : _______________

NB-109

November-2021

B.Sc., Sem.-V

CC-301 : Mathematics
(Linear Algebra – II)

Time : 2 Hours] **[Max. Marks : 50**

Instructions : (1) Attempt any **THREE** questions in Section-I.

(2) Section-II is a compulsory section of short questions.

(3) Notations are usual everywhere.

(4) The right hand side figures indicate marks of the sub question.

SECTION – I

Attempt any **THREE** of the following questions :

1. (a) If T : U $\rightarrow$ V is linear map, $v_0 \in R(T)$ and if $T(u) = \overline{0}_v$ a non-trivial solution $u \neq \overline{0}_u$ then prove that the non-homogeneous operator equation (NH) namely, $T(u) = v_0$ has an infinite number of solutions and if $u_0 \in U$ is a solution of (NH), then $u_0 + N(T)$ is the solution set of (NH). **7**

   (b) If a linear map $T : V_3 \rightarrow V_3$ is defined as

   $T(e_1) = e_1 + e_2$, $T(e_2) = e_2 + e_3$, $T(e_3) = e_1 + e_2 + e_3$, then solve the operator equation $T(x_1, x_2, x_3) = (1, 4, 4)$. **7**

2. (a) If A is a non-empty subset of a real vector space V, then prove that Å (the Annihilator of A) is a subspace of the dual space V*. **7**

   (b) Find the dual basis of the basis B = {(0,1,1), (1,0,1), (1,1,0)} for the vector space V_3. **7**

3. (a) State and derive the Cauchy-Schwarz inequality. **7**

(b) If A = $\{v_1, v_2, ..., v_k\}$ is an orthogonal subset of an inner product space V, then prove that $\| \sum_{i=1}^{k} v_i \|^2 = \sum_{i=1}^{k} \| v_i \|^2$. **7**

4. (a) Prove that every orthogonal set of non-zero vectors is always linearly independent in an inner product space. **7**

(b) Apply the Gram-Schmidt orthogonalization process to the basis B = {(1, 2) (3, 4)} in order to get the orthonormal basis for V_2. **7**

5. (a) If det : $V^n \to R$ is a function satisfying the expected properties of the determinant, then prove the followings :

(i) $\det (v_1, v_2, ..., v_i, ..., v_j, ..., v_n) = - \det (v_1, v_2, ..., v_j, ..., v_i, ..., v_n)$.

(ii) $\det (v_1, \bar{0}, v_2, ..., v_n) = 0$. **7**

(b) If $A = \begin{pmatrix} 1 & 0 & 2 & 0 \\ 1 & 1 & 0 & -1 \\ 1 & 0 & 2 & 1 \\ 0 & 2 & 1 & -1 \end{pmatrix}$ then Compute *det*A without expansion. **7**

6. (a) State and prove the Cramer's rule for solving a system of linear equations. **7**

(b) Use the Laplace expansion about second row to find *det*A if $A = \begin{bmatrix} 0 & -1 & 3 \\ 2 & 5 & -4 \\ -3 & 7 & 1 \end{bmatrix}$ **7**

7. (a) Express the characteristic equation of 2 × 2 matrix A in terms of its trace and determinant.

Also prove that a 2 × 2 real and symmetric matrix has only real eigen values. **7**

(b) Discuss the existence of eigen value and eigen vector of the linear map $T : V_2 \to V_2$ defined as $T(x_1, x_2) = (x_2, x_1)$ for $(x_1, x_2) \in V_2$. **7**

8. (a) If v_1 and v_2 are eigen vectors corresponding to two distinct eigen values λ_1 and λ_2 of a symmetric linear map $T : V \to V$ then prove that v_1 and v_2 are orthogonal vectors of V. **7**

(b) Diagonalize the matrix $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$

Also find the modal matrix which diagonalizes A. **7**

SECTION – II

9. Answer any **FOUR** of the followings in SHORT : **8**

(i) Define a linear functional and the Dual Space of a vector space.

(ii) Define homogeneous and non-homogeneous operator equations.

(iii) Define a Euclidian Space and a Unitary space.

(iv) Define an orthogonal linear map and orthogonal complement of a subspace in an inner product space V.

(v) Without expansion find *det*A if $A = \begin{bmatrix} 3 & 0 & 0 \\ 1 & 2 & 0 \\ 2 & 1 & 3 \end{bmatrix}$

(vi) Define a symmetric linear map and a quadric.
