Seat No. : _______________

DI-101

December-2021

B.Sc., Sem.-III

CC-201 : Statistics
(Distribution Theory – I)

Time : 2 Hours] **[Max. Marks : 50**

Instructions : (1) There are **two** sections in this question paper.
(2) **All** questions in Section – I carry equal marks.
(3) Attempt ANY **THREE** questions from Section – I.
(4) Section – II is compulsory.
(5) Figures to the right indicate full marks of the questions/sub-questions.

Section – I

1. (a) In usual notations, derive the probability mass function of Binomial distribution. **7**
   (b) If a random variable X~Po (m), in usual notations; derive the moment generating function of X. Also, state the cumulant generating function of X. **7**

2. (a) What is Truncation ? Derive truncated poisson distribution. Also, obtain its variance. **7**
   (b) In usual notations, show that the recurrent relation for the cumulants of binomial distribution is **7**

$$K_{r+1} = pq\left(\frac{dk_r}{dp}\right), r = 1, 2, 3, ...$$

3. (a) If a probability density function a random variable X is **7**

$$f(x) = \begin{cases} \frac{1}{\beta(m, n)} x^{m-1}(1-x)^{n-1}, & 0 < x < 1 \\ 0 & , \text{otherwise} \end{cases}$$

   Then, identify the probability distribution of X and obtain its mean and variance.
   (b) A random variable X an exponential distribution with parameter α, in usual notations, show that $V[X] = \frac{1}{\alpha^2}$ **7**

4. (a) For beta type II distribution, derive its mean and harmonic mean. **7**

(b) A random variables X and Y are independent gamma variates with parameters (α, β) and (a, λ) respectively, then show that $Z = X / (X + Y)$ follows beta distribution of first kind. **7**

5. (a) What is Jacobian of transformation? State its uses in probability distribution theory. **7**

(b) If X and Y are independent random variables, in usual notations, derive the probability density function of $U = X\,Y$. **7**

6. (a) If the cumulative distribution X is F(X), then, obtain the cumulative distribution function and probability function of (i) $Y = X+1$, (ii) $Y = X^2$. **7**

(b) Let X be a continuous random variable with probability density function $f(x)$. If $Y=g(x)$ is monotonically increasing or decreasing function of X, then, the probability density function of Y is $h(y) = f(x)\left|\frac{dx}{dy}\right|$. **7**

7. (a) Define order statistics. Derive probability density function of the smallest order statistics. **7**

(b) For uniform distribution with the probability density function of random variable X is $f(x) = \begin{cases} 1 & ,0 < x < 1 \\ 0 & ,\text{otherwise} \end{cases}$

If a random sample (X_1, X_2, X_3) of size 3 is taken on X, derive probability distribution of the smallest order statistics (Y_1). Also, find $P[\,Y_1 < 0.3]$. **7**

8. (a) Derive probability distribution of sample range of order statistics. **7**

(b) In usual notations, derive the probability density function of the largest order statistics (Y_n). **7**

Section – II

9. Attempt ANY **EIGHT** from following : **8**

(1) State one use of binomial distribution.

(2) State mean and variance of Bernoulli distribution.

(3) Give the name of distribution of sum of n independent Bernoulli variates.

(4) Give One applications of poisson distribution.

(5) State the additive property of poisson distribution.

(6) State probability density function of beta type I distribution..

(7) If X~G(a, m) and Y~G(a, n) be two independently distributed gamma variates, then state the distribution of X + Y.

(8) State skewness of a random variable X~G(α, β).

(9) State mean and variance of rectangular distribution.

(10) State joint probability density function of U=U(x, y), and V = V(x, y), given the joint probability density function of(X, Y).