Seat No. : _______________

ML-113

May-2022

B.Sc., Sem.-V

305 : Mathematics
(Discrete Mathematics)

Time : 2 Hours] **[Max. Marks : 50**

Instructions : (1) Attempt any **3** out of first **6** questions. **7th** question is compulsory.
(2) Figures to the right indicate full marks of the question/sub-question.
(3) Notations used in this question paper carry their usual meaning.

SECTION – I

1. (a) Let (L, $\leq$) be a lattice. For any a, b, c $\in$ L, prove that **7**

$$b \leq c \Rightarrow \begin{cases} a * b \leq a * c \\ a \oplus b \leq a \oplus c \end{cases}$$

(b) Let X = {a, b, c, d}. Which of the following relations are partial order relations ? Justify. **7**

(1) $R_1 = \{< a, a >, < b, b >\}$

(2) $R_2 = \{< a, a >, < b, b >, < c, c>, < d, d >\}$

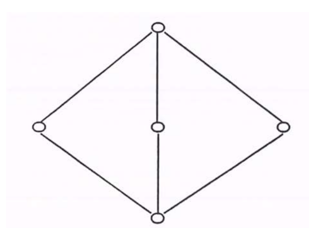
(3) $R_3 = \{< a, b >, < b, a >\}$

2. (a) For a lattice $\langle L, \leq \rangle$ prove that **7**

$a \leq b \Leftrightarrow a * b = a \Leftrightarrow a \oplus b = b$.

(b) Show that $\langle N, D \rangle$ is a poset but not a chain. **7**

3. (a) Define Complemented Lattice and find complement of each element of the following lattice. **7**



(b) Show that $\langle S_{30}, *, \oplus \rangle$ i.e. $\langle S_{30}, D \rangle$ and $\langle P(X), \cap, \cup \rangle$ lattices are isomorphic, where X = {a, b, c}. **7**

4. (a) Let $\langle L, *, \oplus \rangle$ be a distributive lattice. Prove that for any a, b, c $\in$ L, (a * b = a * c) and (a$\oplus$b = a$\oplus$c) $\Rightarrow$ b = c. **7**

(b) Let $\langle P(X)< \cap, \cup \rangle$ be a lattice where X = {a, b, c}. Determine which of the following subsets of P(X) are sublattices. **7**

(1) $S_1 = \{\phi, \{a\}, \{b\}\}$

(2) $S_2 = \{\phi, \{a\}, \{b\}, \{a, b\}\}$

5. (a) State and prove Stone's representation theorem. **7**

(b) Find POS and SOP canonical forms of the Boolean expressions **7**

(1) $\alpha(x_1, x_2, x_3) = (x'_1 * x_2) \oplus x_3$

(2) $\alpha(x_1, x_2, x_3) = x_1 \oplus (x_2 * x_3)$

6. (a) Prove that the product of two distinct minterms is zero. **7**

(b) Let $\langle B, *, \oplus, ', 0, 1 \rangle$ be a Boolean algebra, A be the set of all atoms of B and $x_1, x_2 \in$ B. Prove that **7**

(1) $A(x') = A - A(x)$

(2) $A(x_1 * x_2) = A(x_1) \cap A(x_2)$

SECTION – II

7. Answer in short : (Any **4**) **8**

(1) Define symmetric relation and give an example.

(2) Draw the Hasse diagram of $\langle S_6, D \rangle$

(3) Define Boolean isomorphism.

(4) Find all atoms of $\langle S_{15}, D \rangle$ Boolean algebra.

(5) Show that 0 is the only complement of 1 in a lattice.

(6) Show that $\langle S_{21}, D \rangle$ is a complemented lattice.

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ML-113

May-2022

B.Sc., Sem.-V

305 : Mathematics
(Number Theory)

Time : 2 Hours] **[Max. Marks : 50**

Instructions : (1) Notations are usual everywhere.

(2) Figures to the right indicate marks of the question.

(3) Attempt ANY **THREE** questions of Section – I from Q. **1** to Q. **6**.

(4) Q. **7** of Section-II is compulsory.

SECTION-I

Attempt Any **THREE** :

1. (a) Show that the linear Diophantine equation $ax + by = c$ has solution if $d \mid c$, where $d = \gcd(a, b)$. Also if x_0 and y_0 are any solution of the equation then any other solution is given by $x = x_0 + \left(\frac{b}{d}\right)t$, $y = y_0 - \left(\frac{a}{b}\right)t$, where $t \in \mathbb{Z}$. **7**

   (b) Using Euclidean Algorithm, find the integers x, y, z satisfying $\gcd(198, 288, 512) = 198x + 288y + 512z$ **7**

2. (a) Prove : For given integers a & b with $b > 0$, there exist unique integers q & r such that $a = qb + r$, $0 \leq r < b$. **7**

   (b) Prove that there are an infinite number of primes of the form $4n + 3$. **7**

3. (a) Show that every positive integer $n > 1$ can be expressed as a product of primes in unique way (does not matter of the orders of the prime factors). **7**

   (b) Prove : $p \geq q \geq 5$ & p, q primes $\Rightarrow 24 \mid p^2 - q^2$ **7**

4. (a) Prove that there is an infinite number of primes of the form $4k + 3$. **7**

   (b) Using Chinese Remainder Theorem, solve : $17x \equiv 9 \pmod{276}$. **7**

5. (a) Show that the quadratic congruence $x^2 + 1 \equiv 0 \pmod{p}$, p = prime > 2 has solution if and only if $p \equiv 1 \pmod{4}$ **7**

(b) Prove : $7 \mid 5^{2n} + 3 \cdot 2^{5n-2}, \forall\, n \geq 1$. **7**

6. (a) Prove : If p is a prime then $(p-1)! \equiv -1 \pmod{p}$.

Also by Wilson's theorem verify that 4(29!) + 5! is divisible by 31 or not. **7**

(b) State the Euler's Theorem and prove that $a^{1729} \equiv a \pmod{1729}$, $a \in Z$. Is 1729 pseudo prime or absolute pseudo prime or both ? **7**

SECTION – II

7. Attempt any **four** of the following questions in short : **8**

(a) What is the unit digit of 3^{3^3} ?

(b) Find a prime P such that the numbers $p^2 + 8$ and $p^3 + 4$ are also primes.

(c) State the Well Ordering Principle and the fundamental theorem of arithmetic.

(d) Define prime and relatively prime.

(e) Define Euler's Phi-function.

(f) If $ax \equiv ay \pmod{n}$ and $(a, n) = 1$, then show that $x \equiv y \pmod{n}$.