

MK-106

May-2022

B.Sc., Sem.-V

CC-304 : MATHEMATICS

(Mathematical Programming)

Time : 2 Hours]

[Max. Marks : 50

- Instructions :**
- (1) All questions in Section-I carry equal marks.
 - (2) Attempt any **THREE** questions in Section-I.
 - (3) Question-9 in Section-II is **COMPULSORY**.

SECTION – I

1. (A) Prove that the intersection of two convex set is a convex set. Is union of two convex set is convex ? Justify.
- (B) The manager of an oil refinery must decide on the optimal mix of two possible blending processes of which the input and output per production run are given as follows :

Process (units)	Input (units)			Output (units)	
	Crude A	Crude B		Gasoline X	Gasoline Y
1	5	3		5	8
2	4	5		4	4

The maximum amount available of crude A and B are 200 units and 150 units respectively. Market requirements show that at least 100 units of gasoline X and 80 units of gasoline Y must be produced. The profit per production run from process 1 and process 2 are ₹ 300 and ₹ 400 respectively. Formulate this problem as a linear programming model.

2. (A) Prove that a necessary and sufficient condition for a set S to be convex is that every convex linear combination of points in S belongs to S.
- (B) A firm manufactures headache pills in two sizes A and B. Size A contains 2 grain of aspirin, 5 grain of bicarbonate and 1 grain of codeine. Size B contains 1 grain of aspirin, 8 grain of bicarbonate and 6 grain of codeine. It is found by users that it requires at least 12 grains of aspirin, 74 grains of bicarbonate and 24 grains of codeine for providing immediate effect. It is required to determine the least number of pills a patient should take to get immediate relief. Formulate the problem as a standard LPP.

3. (A) Explain Simplex algorithm for solving linear programming problem.
- (B) Solve the following L.P.P. by simplex method.

$$\text{Min. } Z = x_1 - 3x_2 + 2x_3$$

$$\text{s.to c. } 3x_1 - x_2 + 3x_3 \leq 7, -2x_1 + 4x_2 \leq 12, -4x_1 + 3x_2 + 8x_3 \leq 10, x_1, x_2 \geq 0.$$

4. (A) Explain Gomory's cutting plane method for solving Integer programming problem.
- (B) Using appropriate simplex method to solve :

$$\text{Max } Z = 2x_1 + 3x_2 + 5x_3$$

$$\text{Subject to } 3x_1 + 10x_2 + 5x_3 \leq 15 ; 33x_1 - 10x_2 + 9x_3 \leq 33; x_1 + 2x_2 + x_3 \geq 4 ; x_1, x_2, x_3, \geq 0.$$

5. (A) Prove that the value of the objective function Z for any feasible solution of the primal is not less than the value of the objective function Z* for any feasible solution of the dual.

- (B) Using Duality to solve the following Linear Programming Problem.

$$\text{Max } Z = 3x_1 + 2x_2$$

$$\text{S. to C. } 2x_1 + x_2 \leq 5 ; x_1 + x_2 \leq 3 ; x_1, x_2 \geq 0$$

6. (A) Prove that the dual of the dual is primal. Also explain dual simplex method for solving Linear Programming Problem.
 (B) Using dual simplex method to solve the following Linear Programming Problem.

$$\text{Max } Z = -3x_1 - x_2$$

$$x_1 + x_2 \geq 1 ; 2x_1 + 3x_2 \geq 2 ; \text{ and } x_1, x_2 \geq 0$$
7. (A) Prove that transportation problem has a triangular basis.
 (B) Explain loop in Transportation problem.
8. (A) Explain Hungarian method to solve Assignment Problem.
 (B) Solve the following Transportation problem.

	Destination				Supply	
Source		D ₁	D ₂	D ₃	D ₄	
A		5	3	6	4	30
B		3	4	7	8	15
C		9	6	5	8	15
Demand		10	25	18	7	60

SECTION – II

9. Answer in Short : (Attempt any **FOUR**)
- Define Extreme Point.
 - Give two examples of non-convex set.
 - Define Surplus variable.
 - How we solve any Linear Programming Problem using Two-Phase simplex method ?
 - How we get Primal solution without solving it ?
 - How we solve unbalanced transportation problem ?

