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1205E223

Candidate's Seat No : ________

MSc Sem.-3 Examination

503 EA

Mathematics (Adv Calculus) (New)

May 2022

Time : 2-00 Hours] **[Max. Marks : 50**

Instructions:

1. All the questions in **Section-I** carry equal marks.
2. Attempt any **Three** questions in **Section-I**.
3. Questions in **Section-II** are **COMPULSORY**.

Section-I

1. (A) Find $\partial_x w$ and $\partial_y w$ in terms of the partial derivatives $\partial_1 f$, $\partial_2 f$, and $\partial_3 f$, where $w = f(2x - y^2,\ x \sin 3y,\ x^4)$. 7

   (B) Find the extreme values of $f(x,y) = 3x^2 - 2y^2 + 2y$ on the set $\{(x,y) : x^2 + y^2 \leq 1\}$.

2. (A) Find and classify the critical points of the function $f(x,y) = xy(12 - 3x - 4y)$. 7

   (B) Find the Taylor polynomial of order 4 based at $a = (0,0)$ for the function $f(x,y) = x\sin(x+y)$. 7

3. (A) Investigate the possibility of solving the equations 7

$$\begin{cases} xy + 2yz - 3xz = 0 \\ xyz + x - y = 1 \end{cases}$$

   for two of the variables as functions of the third near the point $(x,y,z) = (1,1,1)$.

   (B) Let $\mathbf{F}(x,y) = x^2 + 6y^2 - 6$.
   Determine whether the set $S = \{(x,y) : \mathbf{F}(x,y) = 0\}$ is a smooth curve. Draw a sketch of S. Examine the nature of S near any point $\Delta\mathbf{F} = 0$. Near which points of S is the graph of a function $y = f(x)$? $x = f(y)$? 7

4. (A) Find an equation of the tangent plane to the following parametrized surface at the point $(1,-2,1)$.
   $x = e^{u-v},\ y = u - 3v,\ z = \frac{1}{2}(u^2 + v^2)$ 7

   (B) Let $(u,v) = f(x,y) = (e^x \cos y, e^x \sin y)$. 7
   Compute the Jacobian $\det Df$. Draw a sketch of the images of some of the lines $x =$ constant and $y =$ constant in the $uv-$plane. Find a formula for a local inverse of f

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5. (A) Find the volume of the region in $\mathbb{R}^3$ above the trianle T in the xy−plane with vertices $(0,0)$, $(1,0)$ and $(1,2)$ and below the surface $z = xy + y^2$. 7

(B) Find the centroid of the tetrahedron bounded by the coordinate planes and the plane $\frac{x}{3} + \frac{y}{5} + \frac{z}{7} = 1$. 7

6. (A) An object with mass density $\rho(x, y, z) = xy$ occupies the cube $\{(x, y, z) : 0 \leq x, y, z \leq 2\}$. Find its mass and center of mass. 7

(B) Find the volume of the region inside both the sphere $x^2 + y^2 + z^2 = 4$ and the cylinder $x^2 + y^2 = 1$. 7

7. (A) Compute $\int_C \mathbf{F} \cdot d\mathbf{x}$,

where $\mathbf{F}(x, y) = (x^2y, x^3y^2)$ and C is the closed curve formed by the portions of the line $y = 4$ and the parabola $y = x^2$, oriented counterclockwise. 7

(B) Let S be the annulus $1 \leq x^2 + y^2 \leq 4$.

Compute $\int_{\partial S} (xy^2dy - x^2ydx)$ by using the Green's theorem. 7

8. (A) Compute $\iint_C (x^2 + y^2)dA$,

where S is the portion of the sphere $x^2 + y^2 + z^2 = 4$ with $z \geq 1$. 7

(B) Evaluate $\iint_S \mathbf{F} \cdot \mathbf{n}\, dA$, 7

where $\mathbf{F}(x, y, z) = x^2\mathbf{i} + z\mathbf{j} - y\mathbf{k}$ and S is the unit sphere $x^2 + y^2 + z^2 = 1$, oriented so that the normal points upward.

Section-II

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(1) The directional derivative of the function $f(x, y) = xy^2 + \sin \pi xy$ at the point $(1, -2)$ in the direction $(\frac{4}{5}, \frac{3}{5})$ is

(A) $\frac{1}{5}(4 + 5\pi)$

(B) $\frac{1}{5}(4 - 5\pi)$

(C) $-\frac{1}{5}(4 + 5\pi)$

(D) $-\frac{1}{5}(4 - 5\pi)$

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(2) The tangent plane to the surface $z = x^2 - y^3$ at the point $(2, -1, 5)$ is

(A) $4x - 3y - z = 6$ (C) $4x - 3y - z = -6$

(B) $4x + 3y - z = 6$ (D) $4x + 3y + z = -6$

(3) The curve $\mathbf{f}(s) = (s^3 - 1, s^3 + 1)$ is a

(A) circle (C) ellipse

(B) parabola (D) straight line

(4) Let $F(x, y) = (x - 1)y(x + y)$.
Let $S = \{(x, y) : F(x, y) = 0\}$.
Near which of the points given below is S the graph of a function $y = f(x)$?

(A) $(1, 0)$ (C) $(1, 1)$

(B) $(2, -2)$ (D) $(0, 0)$

(5) Let $f_1(x, y, z) = x - y + z$, $f_2 = x^2 + y^2 + z^2 - 2xy$, $f_3(x, y, z) = y - x + z$. The functional relation between the functions is

(A) $f_2 = \frac{1}{4}(f_1 + f_3)^2 + \frac{1}{4}(f_1 - f_3)^2$ (C) $f_2 = 2f_1 - f_3$

(B) $f_3 = \frac{1}{4}(f_1 + f_2)^2 + \frac{1}{4}(f_1 - f_2)^2$ (D) $f_2 = \frac{1}{2}(f_1 + f_3)^2 + \frac{1}{2}(f_1 - f_3)^2$

(6) The arc length of the parametrized curve $\mathbf{g}(t) = (3\cos t, 3\sin t, 4t), 0 \le t \le 2\pi$, is

(A) 5π (B) 2π (C) 10π (D) 20π

(7) Let $\mathbb{F}(x, y, z) = (x^2 z,\ 4xyz,\ y - 3xz^2)$. Then curl $\mathbb{F}$ equals

(A) $(2z + 3,\ 0,\ -2x)$ (C) $(-2z,\ 0,\ 6x)$

(B) $(4z + 3, 0, 6x)$ (D) $(1 - 4xy,\ 3z^2 - x^2,\ 4yz)$

(8) Let $\mathbb{F}(x, y, z) = (xy^2,\ xy,\ xy)$. Then div $\mathbb{F}$ equals

(A) $x + y^2$ (C) $x + z + y^2$

(B) $2x + y^2$ (D) $z + x^2$