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1105E175

Candidate's Seat No :_____

MSc Sem.-3 Examination
502
Mathematics
May 2022

Time : 2-00 Hours] **[Max. Marks : 50**

1. Each the question in **Section-I** carry equal 14 marks.
2. Attempt any **Three** questions from **Section-I**
3. Questions in **Section-II** are **COMPULSORY**

Section-I

1. (A) Define the group S_3. What is the centre $Z(S_3)$ of the group S_3? [7]

   (B) Define a cyclic subgroup of a group G. How many generators does the group $\mathbb{Z}_{20}$ have? [7]

2. (A) Give with details an example of an infinite non-Abelian group. [7]

   (B) State (without proof) the fundamental theorem of finite Abelian groups. How many Abelian groups of order 50 are there? Justify. [7]

3. (A) What is the order of the permutation $\alpha = (12)(2345)$ in the group A_{10}? [7]

   (B) Define simple groups. Determine the values of n for which the group $\mathbb{Z}_n$ is simple. [7]

4. (A) Define an automorphism. Prove that $Aut(\mathbb{Z}_n)$ is isomorphic to $U(n)$. [7]

   (B) Define a normal subgroup. Give an example of group G and its subgroup H such that H is not normal in G. [7]

5. (A) Prove or disprove: The group $(\mathbb{R}, +)$ is isomorphic to the group $(\mathbb{Q}, +)$. [7]

   (B) Let $|G| = 60$ and H be a subgroup of G. write down all the possible orders of H? [7]

[P.T.O]

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6. (A) Define homomorphism. If $\phi : G \to G'$ is a homomorphism then prove that ϕ sends the identity to the identity.. [7]

   (B) Define simple groups. If $|G| = 11$. show that G is simple. [7]

7. (A) What is the order of the group $U(50)$? Explain. [7]

   (B) Define the conjugacy class $cl(a)$ of the element $a \in G$.
   When does $cl(a) = \{a\}$ hold for all $a \in G$? Explain. [7]

8. (A) How many homomorphism are there from the group $\mathbb{Z}$ to the group $\mathbb{Z}$? [7]

   (B) How many elements of order 2 are there in the group $\mathbb{Z}_{100} \oplus \mathbb{Z}_{200}$? [7]

Section -II

[8]

1. Any infinite cyclic group G is isomorphic to the group ____

   (A) $(\mathbb{R}, +)$
   (B) $(\mathbb{Z}, +)$
   (C) $(\mathbb{Q}, +)$
   (D) $(\mathbb{Z}_n, +_n)$

2. Let a be any element of group G. Then $|a| = |a^{-1}|$

   (A) Always true.
   (B) True, if G is cyclic.
   (C) True, if G is finite
   (D) True, if G is Abelian.

3. What is the order of the group $G = \mathbb{Z}_7 \oplus U(7)$?

   (A) 42
   (B) 24
   (C) 52
   (D) 34

E-175-3

4. How many elements of the group A_5 have order 3?

(A) 1 (C) 15

(B) 20 (D) 24

5. What is the order of the center $Z(S_3)$ of the group S_3?

(A) 6 (C) 1

(B) 4 (D) 2

6. Which of the following groups is simple?

(A) S_3 (C) $(\mathbb{Z}_4, +_4)$

(B) $(\mathbb{Z}, +)$ (D) $(\mathbb{Z}_7, +_7)$

7. For which values of m and n, the group $G = \mathbb{Z}_n \oplus \mathbb{Z}_m$ is cyclic?

(A) $m = 10, n = 15$ (C) $m = 12, n = 21$

(B) $m = 10, n = 21$ (D) $m = 4, n = 28$

8. What is the order of the group $Aut(\mathbb{Z}_8)$?

(A) 10 (C) 4

(B) 1 (D) 2

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