Seat No. : _______________

JH-124

June-2022

M.Sc., Sem.-II

410 : Mathematics

(Partial Differential Equations)

Time : 2 Hours] **[Max. Marks : 50**

PART – A

Attempt any **THREE** questions : **42**

1. (a) Find the complete integrals of the given partial differential equation with Charpit Method :

   $2(z + xp + yq) = yp^2$

   (b) Eliminate the parameters a and b from the following equation and find the corresponding partial differential equation

   $z^2(1 + a^3) = 8(x + ay + b)^3$

2. (a) Find the complete integral of $2zx - px^2 - 2qxy + pq = 0$

   (b) Find the general integral of $(z^2 - 2yz - y^2)p + x(y + z)q = x(y - z)$

3. (a) Find the complete integral of $pxy + pq + qy = yz$.

   (b) Solve the partial differential equation $z_x z_y = z$ subject to the condition $z(s, -s) = 1$

4. (a) Solve the following equation by Jacobi's Method

   $Z^3 = pqxy$

   (b) Find a solution of $z = p^2 - q^2$ which passes through the curve C with the equation $4z + x^2 = 0$, $y = 0$

5. (a) Using Jacobi's method, find the complete integral of

   $$p_3x_3(p_1 + p_2) + x_1 + x_2 = 0$$

   (b) Find the integral surface of the equation $pq = z$ passing through

   $$C : x_0 = 0,\ y_0 = 0,\ z_0 = s^2$$

6. (a) State the problem of the wave equation in the case of vibrations of a string of finite length and solve it using the method of separation of variables. Consider that both ends are fixed and initial displacement distribution is $f(x)$ and initial velocity distribution is $g(x)$.

(b) State and solve the heat conduction problem for a finite rod of length l with initial temperature distribution in the rod at time $t=0$ given by $f(x)$. Use the method of separation of variables.

7. (a) If u_1 and u_2 are solutions of Neumann's BVP then show that, $u_1 - u_2 = \text{constant}$. Also show that the solution of Neumann's problem is unique upto the addition of a constant.

(b) Solve $u_{tt} - c^2 u_{xx} = F(x, t), 0 < x < l, t > 0,$

$$u(x, 0) = f(x), 0 < x < l,$$

$$u_t(x, 0) = g(x), 0 < x < l,$$

$$u(0, t) = u(1, t) = 0, t > 0,$$

by making use of Duhamel's principle.

8. (a) Solve the following partial differential equation using Fourier Transform

$$u_{tt}(x, t) = c^2 u_{xx}(x, t), x \in (-\infty, \infty), t > 0$$

$$u(x, 0) = f(x), x \in (-\infty, \infty)$$

$$u_t(x, 0) = g(x)\ x \in (-\infty, \infty)$$

with $u(x, t), u_x(x, t) \to 0$ as $x \to \pm\infty, t > 0$

(b) Solve the Initial Boundary Value Problem :

$$u_{tt} = u_{xx}, x \in (0, 1), t > 0,$$

$$u(0, t) = u(1, t) = 0, t > 0$$

$$u(x, 0) = x(1 - x),$$

$$u_t(x, 0) = 0, x \in [0,1]$$

PART – B

9. Choose an appropriate option for the given questions to write the correct answers. (Each carries **one** mark) **8**

(1) Let u(x, t) be the solution of the initial value problem

$u_{tt} - u_{xx} = 0$, $u(x, 0) = x^3$, $u_t(x, 0) = \sin x$. Then, u(π, π)is

(a) π^3 (b) $4\pi^3$

(c) 0 (d) 4

(2) Let f(x, y, z) = c be a one-parameter family of surfaces then choose the necessary condition which is required to form a family of equipotential surfaces

(a) $\frac{\nabla^2 f}{(\nabla f)^2} = -\frac{F''(f)}{F'(f)}$ (b) $\frac{\nabla^2 f}{(\nabla f)^2} = \frac{F''(f)}{F'(f)}$

(c) $\frac{\nabla^2 f}{(\nabla f)^2} = 2\frac{F''(f)}{F'(f)}$ (d) None of these

(3) An artificial node is added for

(a) Dirichlet boundary conditions

(b) Neumann boundary conditions

(c) Forced boundary conditions

(d) Discrete boundary conditions

(4) A two parameter family of solutions is called a complete integral of f(x, y, z, p, q) = 0, if in the region considered, the rank of the matrix

$M = \begin{pmatrix} F_a & F_{xa} & F_{ya} \\ F_b & F_{xb} & F_{yb} \end{pmatrix}$ is

(a) 0 (b) 1

(c) 2 (d) 3

(5) The position of the rod coincides with x-axis, and the rod is _______.

(a) Non-Homogeneous (b) Homogeneous

(c) Non-Linear (d) Semi-Linear

(6) Which of the following equation is elliptic ?

(a) $u_{xx} - u_{yy} - 3u_x + 3u_y = e^{2x+y}$

(b) $3u_{xx} + 2u_{xy} + 5u_{yy} + xu_y = 0$

(c) $u_{xx} - 2u_{xy} + u_{yy} = 0$

(d) $u_{xx} - 2u_{xy} = 0$, for $x > 0$, $y > 0$

(7) For the wave equation the Boundary condition $u(0, t) = 0$ and $u_x(0, t) = 0$ specifies the type

(a) Dirichlet (b) Neumann

(c) Robin (d) Churchill

(8) Which of the following describes the physical phenomenon that is the heat equation ?

(a) $u_{tt} - c^2(u_{xx} + u_{yy} + u_{zz}) = 0$ (b) $\nabla^2 u = f(x, y, z)$

(c) $u_t = K(u_{xx} + u_{yy} + u_{zz})$ (d) $u_{tt} + c^2\nabla^4 u = 0$