Seat No. : _____

JE-102

June-2022

M.Sc., Sem.-II

PHY-407 : Physics

(Quantum Mechanics – II and Mathematical Physics – II)

Time : 2 Hours] **[Max. Marks : 50**

Instructions : (1) **All** questions in Section – **I** carry equal marks.

(2) Attempt any **Three** questions in Section – **I**.

(3) Questions in Section – **II** are COMPULSORY.

Section – I

1. (A) What is the Schrodinger picture ? In the Schrodinger picture, prove that the evolution operator $\hat{U}$ is unitary operator. **7**

   (B) Using the unitary operator $\hat{U}$ in the Schrodinger picture, obtain an equation which is analogue of the Hamilton-Jacobi equation. **7**

2. (A) Prove that the equations of motion for the expectation values in Heisenberg picture and Schrodinger picture are the same. **7**

   (B) Discuss the interaction picture to prove that, the state vector in interaction picture can be determined by Schrodinger equation, while the operators obey the Heisenberg equation. **7**

3. (A) Discuss the creation and annihilation operators. Hence prove that $[\hat{a}, \hat{a}^+] = 1$. **7**

   (B) Discuss why $\hat{a}$ and $\hat{a}^+$ are called the annihilation and creation operators, respectively. **7**

4. (A) Prove why the eigen states of the annihilation operator are called coherent states. **7**

(B) Discuss any three properties of the coherent states. **7**

5. (A) Derive first and second equations of Cauchy's Riemann conditions. **7**

(B) Using Cauchy's Riemann conditions, find out given functions are analytic or non-analytic. **7**

(1) $\cos hz = \frac{e^{z} - e^{-z}}{2}$

(2) $f(Z) = \frac{z}{z^{2}+1}$; where $Z = x + iy$

6. (A) Prove the Residue theorem : **7**

$$\oint_{c} f(Z)\, dZ = 2\pi i(R_1 + R_2 + \ldots\ldots + R_n)$$

(B) (1) Show that $f(Z) = \frac{2z+3}{z+2}$; (where $Z = x + iy$) is non-analytic function. **4**

(2) Show that $f(Z) = \sin z$; (where $Z = x + iy$) is an analytic function. **3**

7. (A) Explain importance of Integral equations in Physics. Also describe various classifications of integral equations. **7**

(B) Convert following differential equation into an integral equation.

$y'' + y = 0$ with $y(0) = -1$ and $y'(0) = -1$. **7**

8. (A) Describe method of conversion of second ordered differential equation into an integral equation. **7**

(B) Obtain Green's function for one dimensional problem. **7**

Section – II

9. Multiple Choice Questions : **8**

1. The operator $\hat{a}^+ \hat{a}$ is also called _____ operator.

   (A) ladder (B) number

   (C) lowering (D) annihilation

2. The eigen value of $\hat{a}^+ \hat{a}$ is _____.

   (A) always positive (B) always zero

   (C) always negative (D) always infinite

3. In _____ picture the state vector Ψ is a function of time, but the dynamical variable $\hat{A}$ is independent of time.

   (A) Heisenberg (B) Schrodinger

   (C) Perturbation (D) Interaction

4. The quantity $< \Psi \mid \hat{A} \mid \Psi >$ represents

   (A) Expectation value *of* $\hat{A}$ for non-normalized wave function Ψ

   (B) Eigen value of $\hat{A}$ for normalized wave function Ψ

   (C) Expectation value of $\hat{A}$ for normalized wave function Ψ

   (D) The norm of $\hat{A}$

5. If $f(Z) = (i-i^2)^3$ then $f(Z^*) =$

   (A) $-2 + 2i$ (B) $-2 - 2i$

   (C) $2 + 2i$ (D) $i - 1$

6. $R_e \sqrt{i} + I_m \sqrt{i} =$

(A) 0, 1 (B) 1, –1

(C) $0, \sqrt{2}$ (D) $1, \sqrt{2}$

7. While solving a partial differential equation using a variable separable method, we equate the ratio to a constant which.

(A) can be positive or negative integer or zero

(B) can be positive or negative rational number or zero

(C) must be a positive integer

(D) must be a negative integer

8. A Green's function is _____

(A) the impulse response of an inhomogeneous linear differential operator

(B) the impulse response of an homogeneous linear differential operator

(C) the impulse response of an inhomogeneous non-linear differential operator

(D) the impulse response of an homogeneous non-linear differential operator