Seat No. : _______________

AL-113

April-2022

B.Sc., Sem.-VI

EC-311 : Mathematics

(Convex Analysis and Probability Theory)

Time : 2 Hours] **[Max. Marks : 50**

Instructions : (i) Notations are usual everywhere.

(ii) Figures to the right indicate marks of the question.

(iii) Attempt ANY **THREE** questions of Section – **I** from Q. **1** to Q. **6**.

(iv) Q. **7** of Section – **II** is **compulsory**.

SECTION – I

1. (a) Define monotonically increasing and decreasing functions on an interval I.

Also show that the function f : R → R defined as $f(x) = x^2$ is monotonically increasing on $[0, \infty)$ and decreasing on $(-\infty, 0]$. **7**

(b) If I is an interval and f : I → R be a strictly monotonic function such that f(I) is an interval then prove that f is one-one and continuous. **7**

2. (a) If I is an interval containing more than one point and f : I → R is a differential function then prove that f is non-negative throughout I ⇔ f is monotonically increasing on I. **7**

(b) Discuss the Monotonicity of the function f : R → R defined as

$$f(x) = x^4 - 8x^3 + 22x^2 - 24x + 7$$ **7**

3. (a) Answer following : **7**

(1) Define Null and Certain Events with one suitable example for each.

(2) What are mutually exclusive and exhaustive events? Give example for it.

(b) State addition rule of probability for two events.

Using the addition rule of probability for two events A and B defined on a finite sample space such that P[A] = 1/3, P[B] = 1/5 and P[both A and B occur simultaneously] = 1/4, then find the probability of following events : **7**

(i) $\overline{A}$ (ii) $\overline{A \cup B}$ (iii) $\overline{A} \cap \overline{B}$ (iv) $\overline{A} \cap B$, if events A and B are independent.

4. (a) (i) Define, Equally likely Elementary event, Classical and axiomatic definitions of probability. **7**

(ii) Define Sample space with its types. Give one example for each of the three terms.

(b) Two balanced dice thrown once, simultaneously. Describe the sample space. Find the probability of the following events: **7**

(i) even number on a first dice and odd on second dice (ii) sum of points on both dice are 7 (iii) sum of numbers on two dice is divisible by 3 (iv) difference of numbers on two dice is divisible by 5.

5. (a) A X follows binomial distribution, with parameters n and p, state probability function of X. Also, if parametric values are $n = 5$ and $p = 1/3$, (i) find mean & variance, $P(X = 1)$, $P(X < 2)$. **7**

(b) For a normal distribution, state its probability density function. Answer, why normal distribution is considered to be symmetric distribution ?

Hence or otherwise, state application of normal distribution. **7**

6. (a) A random variable X follows Poisson distribution with parameter $m = 2$. Then, find $P(X = 1)$, $P(X < 2)$, $P(1 < X < 4)$. **7**

(b) A car hire firm has two cars, which it hires out day by day. The number of demands for a car on each day is distributed as a Poisson with mean 1.5. Calculate the probability of events (i) neither car is used on a particular day (ii) some demands are refused. **7**

SECTION – II

7. Attempt any **four** of the following questions **in short** : **8**

(a) Define Hyper Plane and Convex hull of a set.

(b) State the Intermediate Value Theorem.

(c) Give examples each one of convex and non-convex sets of R^2.

(d) Three coins are tossed, find the probability that exactly three head appears.

(e) What are independent events? If events A and B are independent, then are $\overline{A}$ and B independent ?

(f) In publication house, number of printing mistakes are treated as a random variable X. State the probability distribution X.

Value of e^{-x}

[$e^{-0.5} = 0.607$, $e^{-1} = 0.368$, $e^{-1.5} = 0.223$, $e^{-2} = 0.135$, $e^{-2.5} = 0.082$, $e^{-3.0} = 0.050$, $e^{-3.5} = 0.030$]

Seat No. : _______________

AL-113

April-2022

B.Sc., Sem.-VI

EC-311 : Mathematics

(Operation Research)

Time : 2 Hours] **[Max. Marks : 50**

Instructions : (1) Attempt any **THREE** questions from SECTION – **I**.

(2) SECTION – **II** is **compulsory**.

(3) Right hand side figure indicates marks of that question.

SECTION – I

1. (A) Discuss Economic Order Quantity (EOQ) Model with finite replenishment rate. **7**

(B) A Company uses rivets at a rate of 5000 kg per year, rivets costing ₹ 2 per kg. It costs ₹ 20 to place an order and the carrying cost of inventory is 10% per annum. How frequently should order for rivets be placed and how much ? **7**

2. (A) Discuss Economic Order Quantity (EOQ) model with shortages. **7**

(B) Find the most economic batch quantity of a product on a machine if the production rate of that item on the machine is 200 pieces per day and the demand is uniform at the rate of 100 pieces per day. The ordering cost is ₹ 200 per batch and the cost of holding one item in inventory is ₹ 0.81 per day. Find cycle time and length of production cycle. How will the batch quantity vary if the production rate is infinite ? **7**

3. (A) Write the basic differences between PERT and CPM. **7**

(B) Draw an arrow diagram showing the following relationships : **7**

Activity	A	B	C	D	E	F	G	H	I	J	K	L	M	N
Immediate Predecessor	–	–	–	A, B	B, C	A, B	C	D, E, F	D	G	G	H, J	K	I, L

4. (A) Discuss various steps involved in the applications of PERT and CPM. **7**

(B) An established company has decided to add a new product to its line. It will buy the product from a manufacturing concern, package it, and sell it to a number of Distributors selected on a geographical basis. Market research has indicated the volume expected and the size of sales force required. The steps shown in the following table are to be planned : **7**

Activity	Description	Precedence	Time (Weeks)
A	Organize sales office	–	6
B	Hire salesmen	A	4
C	Train salesmen	B	7
D	Select advertising agency	A	2
E	Plan advertising campaign	D	4
F	Conduct advertising campaign	E	10
G	Design package	–	2
H	Setup packaging facilities	G	10
I	Package initial stocks	H, J	6
J	Order stock from manufacturer	–	13
K	Select distributors	A	9
L	Sell to distributors	K, C	3
M	Ship stocks	I, L	5

(i) Draw an arrow diagram for this project.

(ii) Indicate the critical path.

5. (A) Explain the theory of dominance in the solution of rectangular game. **7**

(B) For the following payoff matrix, transform the zero-sum game into an equivalent linear Programming problem and solve it by using simplex method. **7**

$$\begin{bmatrix} 1 & -1 & 3 \\ 3 & 5 & -3 \\ 6 & 2 & -2 \end{bmatrix}$$

6. (A) Let E(p, q) be such that both $\min\limits_{q} \max\limits_{p} E(p, q)$ and $\max\limits_{p} \min\limits_{q} E(p, q)$ exist, then prove that $\max\limits_{p} \min\limits_{q} E(p, q) \leq \min\limits_{q} \max\limits_{p} E(p, q)$ **7**

(B) Solve the following game whose payoff matrix is given by : **7**

$$\begin{bmatrix} 3 & -1 & 4 & 6 & 7 \\ -1 & 8 & 2 & 4 & 12 \\ 16 & 8 & 6 & 14 & 12 \\ 1 & 11 & -4 & 2 & 1 \end{bmatrix}$$

SECTION – II

7. Attempt any **FOUR** short questions : **8**

(1) Give type of inventory.

(2) Give full form of the notation “ROL”, “R”, “PC”, “A”.

(3) Explain event.

(4) Explain Looping and Dangling.

(5) Explain Maximin and Minimax Principle in short.

(6) Write formula for probabilities of both players and value of the game of matrix method.

AL-113

April-2022

B.Sc., Sem.-VI

EC-311 : Mathematics

(Bio-Mathematics)

Time : 2 Hours] **[Max. Marks : 50**

Instructions : (1) Attempt any **three** questions from Q-**1** to Q-**6**.

(2) Q-**7** is **compulsory**.

(3) Notations are usual, everywhere.

(4) Figures to the right indicate marks of the question/Sub-question.

1. (A) Derive expressions for number of susceptible and number of infected persons at time t considering SI model. **7**

   (B) If contact rate $\alpha = 0.001$, $I_0 = 1$ and $S_0 = 2000$, then determine **7**

   (1) The time at which the rate of appearance of new cases is maximum.

   (2) The maximum rate of appearance of new cases.

2. (A) Discuss SIS model with constant coefficient. **7**

   (B) If $\rho = 100$ and $S_0 = 1000$, find the number of susceptible left by the time 200 infected individuals are removed from circulation. Also find the number of infective if $I_0 = 5$. **7**

3. (A) Derive Euler – Lotka equation. **7**

   (B) Find dominant Eigen value for Fibonacci's rabbits problem. **7**

4. (A) Discuss BLL model. **7**

   (B) Find the long term growth rate of the population modeled by Leslie **7**

   matrix $\begin{bmatrix} 0 & 0 & 8 \\ .25 & 0 & 0 \\ 0 & .5 & 0 \end{bmatrix}$

5. (A) Discuss exponential growth model propounded by Malthus. **7**

(B) A population $x(t)$ is growing according to logistic equation, and $x(t_1) = n_1$, $x(t_1 + T) = n_2$ and $x(t_1 + 2T) = n_3$ then show that the carrying capacity is given by **7**

$$K = \frac{\frac{1}{n_1} + \frac{1}{n_3} - \frac{2}{n_2}}{\frac{1}{n_1 n_3} - \frac{1}{n_2^2}}.$$

6. (A) Derive expression for Logistic growth curve. Hence draw the graph of Logistic growth curve. **7**

(B) In a population of birds, the proportionate birth rate and the proportionate death rate are 0.05 per year and 0.06 per unit time respectively. Further suppose that immigration rate is 3 birds per unit time and there is no emigration. If initial population is given by $x(0) = 100$, then find expression for the population size $x(t)$ at any time t. **7**

7. Attempt any **four** of the following : (Answer in brief) **8**

(1) Write only basics equations of the General Deterministic model with removal (SIR model).

(2) Define maternity function.

(3) List limitations of Logistic model.

(4) Define Latentor incubation period.

(5) In Discrete – Time Discrete – Age – Scale population models: for which values of largest absolute eigen value, population will extinct ?

(6) Define immigration and emigration.