Seat No. : _______________

AI-123

April-2022

B.Sc., Sem.-VI

308 : Mathematics

(Analysis – II)

Time : 2 Hours] **[Max. Marks : 50**

Instructions : (1) **All** Questions in Section – I carry equal marks.

(2) Attempt any **THREE** questions in Section – I.

(3) Question – **9** in Section – II is COMPULSORY

SECTION – I

1. (a) State and prove First Fundamental theorem of Integral Calculus. **7**

(b) Let $f(x) = 3x^2/5$ on $[0, 1]$ for $n \in N P_n = \left\{0, \frac{1}{n}, \frac{2}{n}, \frac{3}{n}, \frac{4}{n} ... \frac{n-1}{n}, 1\right\}$ then find $\lim\limits_{n\to\infty} U[f;P_n]$ and $\lim\limits_{n\to\infty} L[f;P_n]$. **7**

2. (a) If f and g are continuous on [a, b] and if $g(t) \geq 0$ for $a \leq t \leq b$ then prove that there exists $c \in (a, b)$ such that $\int_a^b f(x)\, g(x) dx = f(c) \int_a^b g(x)\, dx$. **7**

(b) Prove that every continuous function on [a, b] is Riemann integrable. **7**

3. (a) Prove that the series $\sum \frac{1}{n!} = 1 + 1 + \frac{1}{2!} + \frac{1}{3!} + ... + \frac{1}{n!} + ...$ converges to the value e, which is an irrational number. **7**

(b) Prove that if $p > 1$, the series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges and if $p \leq 1$, the series diverges. **7**

4. (a) State and prove Cauchy's Root Test. **7**

(b) Test for convergence : **7**

(i) $\sum_{n=1}^{\infty} \frac{n^{1/2}}{3n^{3/2}+1}$ (ii) $\sum_{n=1}^{\infty} \left(1+\frac{2}{n}\right)^{-n^2}$

5. (a) State and prove Mertens' Theorem. **7**

(b) Find the set of convergence (interval of convergence) and radius of convergence for the power series $\sum_{n=1}^{\infty} \frac{n^2(x-2)^n}{(n^2+2)2^n}$. **7**

OR

6. (a) State and prove Leibnitz Alternating Series test. **7**

(b) For the following, determine whether the series converges absolutely, converges conditionally, or diverges : **7**

(i) $\sum \frac{(-1)^n}{(3n^2+4)}$ (ii) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^{1/5}}$

7. (a) State and prove binomial theorem. **7**

(b) Write down Taylor's formula with Lagrange form of remainder for **7**

$f(x) = \log(1+x)$ about $a = 2$ and $n = 4$.

8. (a) Let f be a real valued function on [a, a+h] and $f^{n+1}(x)$ is continuous on[a, a + h]. Then prove that **7**

$$f(x) = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \dots + \frac{f^{(n)}(a)}{n!}(x-a)^n + R_{n+1}(x)$$

for $x \in [a, a+h]$

Where $R_{n+1}(x) = \frac{1}{n!}\int_a^x (x-t)^n f^{(n+1)}(t)dt$.

(b) Find the power series solution of $y' - y + 1 = 0$ with the condition $y(0) = 2$. **7**

SECTION – II

9. Attempt any **four** short questions : **8**

(1) For $f(x) = \cos x$ on $[0, \pi]$. Find $U[f, P]$ for the partition $P = \{0, \pi/2, \pi\}$

(2) Test for convergence : $\int_{2}^{\infty} \frac{1}{\sqrt{x^2 - 1}} \, dx$

(3) Give an example of divergent series Σa_n for which $\Sigma (a_n)^2$ convergent.

(4) Find limit superior and limit inferior of the sequence :

$$\left((-1)^n \left(1 + \frac{1}{n} \right) \right)$$

(5) Write Maclaurin series expansion of $\sin x$

(6) Give an example of the absolute convergent series.

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