Seat No. : _______________

LD-125

April-2014

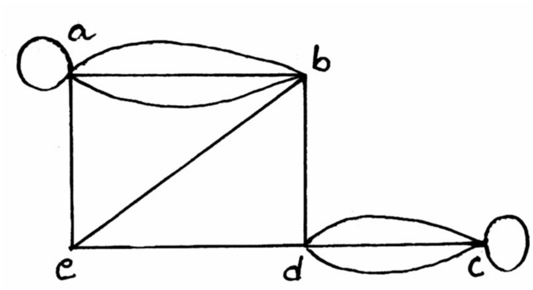
F.Y. M.Sc. (CA & IT) (Sem.-II) Integrated

Matrix Algebra & Graph Theory

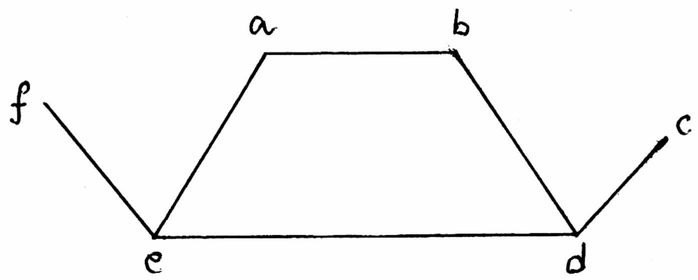
Time : 3 Hours] **[Max. Marks : 100**

1. (A) Attempt any **one :** **8**

(i) Define degree of a vertex and degree sequence of a graph. State and prove the first theorem of graph theory. Verify the same for the graph :



(ii) Given the following graph :



(1) Find its complement.

(2) Find its square.

(3) Is it bipartite ? Justify.

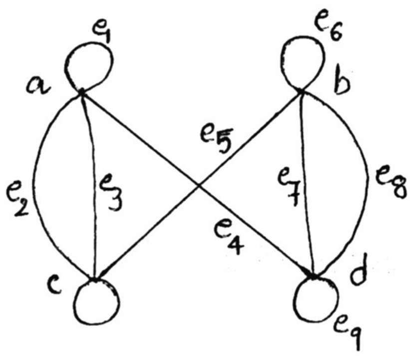
(4) Find the Vertex Deleted sub graph G-d of G.

(B) Attempt any **two :** **8**

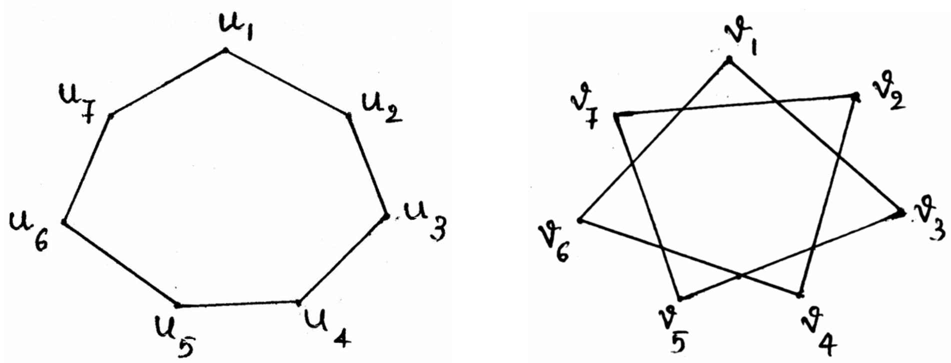
(i) Represent the following graph using :

(1) Adjacency matrix

(2) Incidence matrix



(ii) When are two graphs said to be isomorphic ? Are the following graphs isomorphic ? Justify.



(iii) (1) How many vertices does a full 4-ary tree with 100 leaves have ?

(2) How many leaves does a full 5-ary tree with 100 internal vertices have ?

(C) Attempt **all** : **4**

(i) How many edges does a graph have if its degree sequence is 4, 3, 3, 2, 2 ?

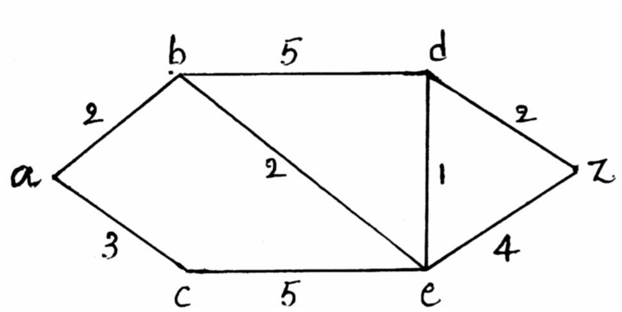
(ii) For what value(s) of m and n is $k_{m,n}$, the complete bipartite graph, a complete graph ?

(iii) Give an example of a self-complementary graph.

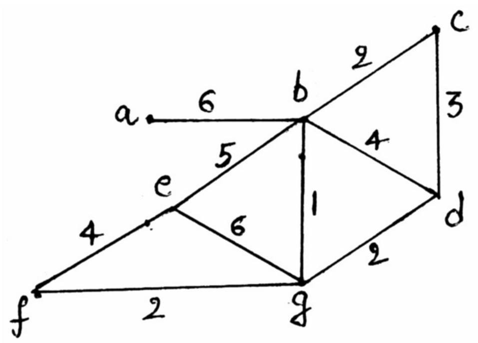
(iv) What is the sum of the entries in a row of the incidence matrix ?

2. (A) Attempt any **one :** **8**

(i) Apply Dijkstra's Algorithm to find the shortest path from 'a' to 'z' in the following graph :



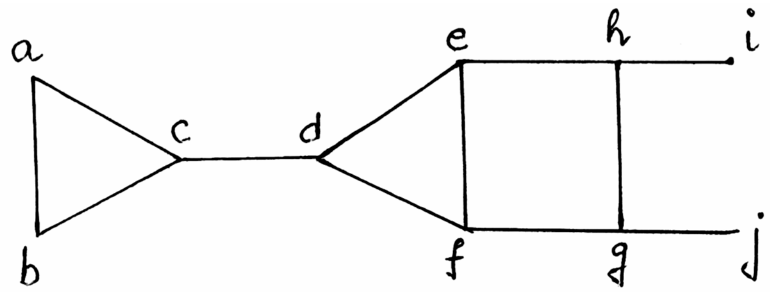
(ii) Differentiate between Kruskal's Algorithm and Prim's Algorithm. Find a minimal spanning tree using Kruskal's and Prim's Algorithm for the following graph :



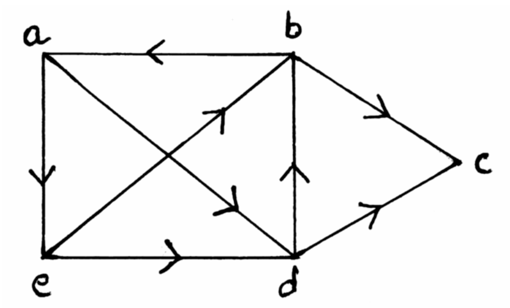
(B) Attempt any **two :** **8**

(i) Let G be an acyclic graph with n-vertices and k-connected components, that is, ω(G) = k. Then prove that G has n-k edges.

(ii) Apply Breadth First Search (BFS) algorithm to find the shortest path from 'a' to 'j' in the following graph :



(iii) What is a strongly connected and a weakly connected graph ? Determine whether the following graph is strongly connected and if not whether it is weakly connected. Justify your answer.



(C) Attempt **all** : **4**

(i) How many different spanning trees does the complete graph K_6 have ?

(ii) True or False : A connected graph with 10 vertices can have 8 edges. Justify.

(iii) Define a bridge.

(iv) Define a spanning tree.

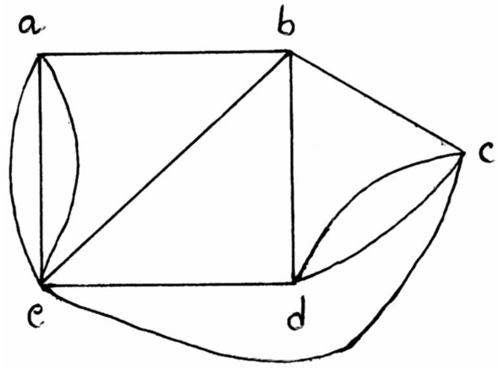
3. (A) Attempt any **one :** **8**

(i) State and prove Euler's formula for planar graphs.

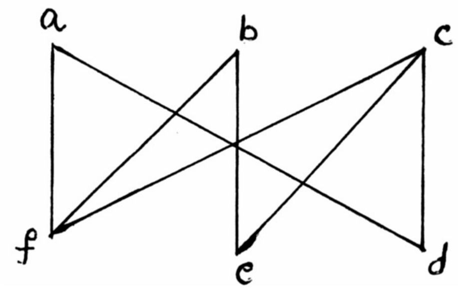
(ii) Prove that the complete graph K_5 is non-planar.

(B) Attempt any **two :** **8**

(i) Define an Euler trail and an Euler tour. Determine whether the given graph has (i) an Euler trail, (ii) an Euler tour. Construct them if they exist.



(ii) Define a plane and a planar graph. Is the following graph planar ? If yes, redraw it without any crossings and also find the number of faces of this graph.



(iii) Find the inverse of the matrix :

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$$

(C) Attempt **all :** **4**

(i) Is K_4 a plane graph ? Is it planar ? Explain.

(ii) What is the crossing number of K_5 ?

(iii) Define a symmetric matrix.

(iv) Define an orthogonal matrix.

4. (A) Attempt any **one :** **8**

(i) Verify the Caley-Hamilton theorem for the matrix

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} \text{ and hence find } A^{-1}.$$

(ii) Define a linear transformation. Let $T : R^3 \to R^2$ be given by $T(x, y, z) = (x + y, y + z)$. Is T a linear transformation ?

(B) Attempt any **two :** **8**

(i) Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 1 & -2 \\ -5 & 4 \end{bmatrix}$

(ii) Find the rank of the matrix $A = \begin{bmatrix} 6 & 1 & 1 & 1 \\ 16 & 1 & -1 & 5 \\ 7 & 2 & 3 & 0 \end{bmatrix}$

(iii) Examine the consistency and if consistent, solve the system of equations

$x + 2y = 3$

$y - z = 2$

$x + y + z = 1$

(C) Attempt **all :** **4**

(i) Determine whether the set of vectors {(1, 0), (0, 1), (0, 0)} is linearly dependent or independent.

(ii) Find the product $[-1 \ \ 3 \ \ 5] \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

(iii) Define linear combination.

(iv) Define Span.

5. (A) Attempt any **one :** **8**

(i) Find the Range, rank, Null Space and nullity of the linear transformation

$T : V_3 \rightarrow V_4$ given by $T(x_1, x_2, x_3) = (x_1, x_1 + x_2, x_1 + x_2 + x_3, x_3)$

(ii) Find the matrix of the linear transformation $T : R^2 \rightarrow R^2$ given by

$T(x, y) = (x + y, x - y)$

$B_1 = \{(1, 0), (0, 1)\}$

$B_2 = \{(1, 2), (3, 4)\}$

(B) Attempt any **two :** **8**

(i) How many solutions are there in $x_1 + x_2 + x_3 + x_4 = 17$ subject to the constraints $x_1 \geq 1, x_2 \geq 2, x_3 \geq 3, x_4 \geq 0$?

(ii) How many committees of five people can be chosen from 15 men and 8 women if (i) Exactly three men must be on each committee, (ii) At least 3 women must be on each committee.

(iii) There are 2504 computer science students at a school. Of these, 1876 have taken a course in Pascal, 999 have taken a course in Fortran and 345 have taken a course in C. Further 876 have taken courses in both Pascal and Fortran, 231 have taken courses in both Fortran and C and 290 have taken courses in both Pascal and C. If 189 of these students have taken courses in Fortran, Pascal and C, how many students have not taken a course in any of these three programming languages ?

(C) Attempt **all :** **4**

(i) Find the number of terms in the expansion of $(3 - 2x + 5z)^7$.

(ii) How many different permutations can be made out of the letters of the word BRAVE ?

(iii) In how many different ways can 7 different beads be arranged to form a necklace ?

(iv) Expand $(a + b)^n$.
