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2504E0090

Candidate's Seat No : __________

M.Phil. Science Examination
Paper-1
Mathematics
April 2019

Time : 2-30 Hours] **[Max. Marks : 70**

Q.1 Attempt any three. 18

(a) Define F_σ set. Find two functions f, g $\mathbb{R} \to \mathbb{R}$ such that $D_f = \mathbb{Z}$ and $D_g = \mathbb{R}$. Give details. (Here, D_f denotes the set of discontinuities of f.)

(b) Define Cesaro -$(C, 1)$ summability of the sequence $\{x_n\}$ of real numbers. Prove that every convergent sequence $\{x_n\}$ is $(C, 1)$ summable to its limit.

(c) Define $(C, 1)$ summability of the sequence $\{x_n\}$. Give an example of a bounded sequence $\{x_n\}$ of real numbers that is not $(C, 1)$ summable.

(d) Define convex function. Determine all the second and third degree polynomials that are convex on $\mathbb{R}$. Explain.

(e) Define convex function. State and prove the generalized AM-GM inequality using convex functions.

Q.2 Attempt any three. 18

(a) Show that the subset G of $\mathbb{R}$, where $G = \{a + b\sqrt{2} : a, b \in \mathbb{Q}, a^2 + b^2 \neq 0\}$, is a group under usual multiplication of two real numbers.

(b) Let $G = \left\{ \begin{bmatrix} a & b \\ -b & a \end{bmatrix} : a, b \in \mathbb{R}, a^2 + b^2 \neq 0 \right\}$. Show that G is a commutative group under matrix multiplication. Show that G is isomorphic to the group of non-zero complex numbers C^*, under multiplication.

(c) Define the set of Mobius transformations and show that they form a group under composition of mappings.

(d) If K is a subgroup of G, show that $W = \cap\{g^{-1}Kg : g \in G\}$ is a normal subgroup of G.

(e) Let G be a finite group of even order. Show that there exists at least one element $a \neq e$ in G such that $a^2 = e$.

Q.3 Attempt any three 18

(a) Determine the eigenvalues and eigenvectors of the linear transformation L on $\mathbb{R}^3$ given by

$$L[x_1 \ \ x_2 \ \ x_3]^T = [x_1 + 2x_2, \ 3x_3, \ x_2 - 2x_3]^T$$

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(b) Determine the solution of the following differential equation by the eigenvector method:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} -1 & 2 \\ -2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}; \quad x(0) = 1, \quad y(0) = 2.$$

(c) Determine the eigenvalues and associated eigenvectors of the following linear operator:

$$L : V_{2\times 2} \to V_{2\times 2} : \begin{bmatrix} a & b \\ c & d \end{bmatrix} \mapsto \begin{bmatrix} d & -c \\ -b & a \end{bmatrix}$$

(d) Find a nonsingular matrix P such that $P^{-1}AP$ is a diagonal form, where A is given by : $\begin{bmatrix} 0 & 2 & 0 \\ 2 & 0 & 2 \\ 0 & 2 & 0 \end{bmatrix}$

(e) Determine the matrix B such that A is congruent to a diagonal matrix D, where A and D are given respectively by $\begin{bmatrix} 1 & 0 & -4 \\ 0 & 5 & 4 \\ -4 & 4 & 3 \end{bmatrix}$, $diag[3, -3, 9]$.

Q.4 Attempt any two. **16**

(a) Determine (i) the right quotient and the right remainder, and (ii) the left quotint and left remainder of $A(\lambda)$ on dividing by $B(\lambda)$ when

$$A(\lambda) = \begin{bmatrix} \lambda^3 & \lambda^2 \\ \lambda & 1 \end{bmatrix}, \; B(\lambda) = \begin{bmatrix} \lambda & 1 \\ 1 & \lambda \end{bmatrix}.$$

(b) Determine the co-gredient transformations $\mathbf{x} = B\mathbf{u}$ and $\mathbf{y} = B\mathbf{v}$ such that the following bilinear form $\mathbf{x}^T A\mathbf{x}$ is congruent to $\mathbf{u}^T D\mathbf{v}$, where D is diagonal:

$$8x_1y_1 + 12x_1y_3 - 2x_2y_2 + 12x_3y_1 - 2x_3y_3$$

(c) Determine the definiteness of the following quadratic form in $\mathbb{R}^3$:

$$2x_1^2 + 4x_2^2 + 8x_3^2 + 2x_1x_2 - 2x_1x_3 - 2x_2x_3$$

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Candidate's Seat No :___________

M.Phil. Science Examination
Paper-2
Mathematics
April 2019

Time : 2-30 Hours] **[Max. Marks : 70**

MAT 602 (Recent Advances in Mathematics: Mathematical Methods and Modelling)

Q.1 (a) 14

(i) A company is negotiating with its union for wages. Positive signs in the table represent wage increase while negative sign represents continue with the current wage structure. What are the optimal strategies for the company as well as union? What is the value of the game?

		Conditional costs to the company (Rs. In lakhs)			
		Union strategies			
Company strategies		U_1	U_2	U_3	U_4
	C_1	0.25	0.27	0.35	- 0.02
	C_2	0.20	0.06	0.08	0.08
	C_3	0.14	0.12	0.05	0.03
	C_4	0.30	0.14	0.19	0.00

(ii) Obtain the optimal strategies for a two-persons and the value of the game for two-persons zero-sum game whose payoff matrix is as follows:

		Player B	
		B_1	B_2
Player A	A_1	1	-3
	A_2	3	5
	A_3	-1	6
	A_4	4	1
	A_5	2	2
	A_6	-5	0

OR

(b)(i) Given payoff table, transform the zero-sum game into an equivalent LPP and solve it using simplex method. 14

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		Player B		
		B_1	B_2	B_3
Player A	A_1	9	1	4
	A_2	0	6	3
	A_3	5	2	8

(ii) In an election campaign, the strategies adopted by the ruling and opposition parties, along with payoffs (ruling party's per cent share in votes polled) are given in table below:

Ruling Party's strategies	Opposition Party's strategies		
	Campaign one day in each city	Campaign two days in large towns	Spend two days in ruler areas
Campaign one day in each city	55	40	35
Campaign two days in large towns	70	70	55
Spend two days in ruler areas	75	55	65

Assume a zero-sum game. Find the optimal strategies for both parties and expected payoff to ruling party.

(c) Choose correct answer 04

(i) The size of the payoff matrix of a game can be reduced by using the principle of

a. game inversion.
b. rotation reduction.
c. dominance.
d. game transpose.

(ii) In a mixed strategy game

a. no saddle point exists.
b. each player selects the same strategy without considering other player's choice.
c. each player always selects same strategy.
d. all of the above.

(iii) When the sum of gains of one player is equal to the sum of looses to another player in a game, the situation is known as

a. biased game.
b. zero-sum game.
c. fair game.
d. all of the above.

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Candidate's Seat No :_________

M.Phil. Science Examination
Paper-3
Mathematics
April 2019

Time : 2-30 Hours] **[Max. Marks : 70**

Q.1) Attempt any **Five**: (All questions carry 7 marks each) [35]

(a) State (only) the Bayes' theorem.
An insurance company believes that people can be divided into two classes: those who are accident prone and those who are not. Their statistics show that an accident-prone person will have an accident at some time within a fixed 1-year period with probability 0.4, whereas this probability is 0.2 for a non accident-prone person. If we assume that 30 percent of the population is accident prone,

(i) what is the probability that a new policyholder will have an accident within a year of purchasing a policy?

(ii) Suppose that a new policyholder has an accident within a year of purchasing a policy. What is the probability that he or she is accident prone?

(b) State and prove the Chebyshev's inequality. State its usefulness.
Given mean and standard deviation as 8 and 1.5 respectively and the shape of the distribution unknown, determine an interval such that the probability is at least 8/9 that an observation will fall within that interval.

(c) Let $f(x) = ke^{-ax}(1 - e^{-ax}),\ \ x, a > 0.$
Find k such that $f(x)$ is a density function. Find the corresponding cumulative distribution function. Find $P(x > 1)$.

(d) The continuous random variable W has the PDF $f_W(w) = 12w^2(1 - w), 0 < w < 1$
Calculate $P(w) < \frac{1}{2}$ and determine an expression for the CDF $F_W(w)$.
Also calculate $E(W), Var(W)$.

(e) Determine the moment generating function of the two parameter random variable X, defined by the probability density function $f(x) = \lambda e^{-\lambda(x-\alpha)}, x \geq \alpha\ ;\ \lambda, \alpha > 0$. Determine the mean and variance of X.

(f) State the pdf of a Poisson distribution. State its application. In standard notation, develop the Poisson distribution as a limiting form of the Binomial distribution.

(g) Consider the following probability density function

$$f_X(x) = \begin{cases} 4x & 0 \leq x \leq 1/2 \\ 4(1-x) & 1/2 \leq x \leq 1 \\ 0 & otherwise \end{cases}$$

Show that it is indeed a pdf. Obtain the distribution function $F_X(x)$

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Q.2] Attempt any **Five**: (All questions carry 7 marks each) [35]

(a) For a random variable that follows the binomial distribution, find the first and the second moments about the origin and the second central moment. For a random variable that follows the gamma distribution, find the first and the second moments about the origin.

(b) In standard notations, derive the Mean and Variance of the Normal distribution.

(c) Define Moment Generating Function MGF. Explain how to use MGF to find the moments. Using it, find the mean and variance of a random variable X, with

$$M_X(t) = \left(1 - \frac{t}{5}\right)^{-1} \quad , t < 5$$

(d) Give the PDF of binomial distribution. Find its MGF. Using it, find the mean and variance

(e) Explain the reproductive property of the Normal Distribution.
An assembly consists of three linkage components X_1, X_2 and X_3 in series. Let $Y=X_1+X_2+X_3$. The properties of X_1, X_2 and X_3 are given below, with means in centimetres and variance in square centimetres.

$$X_1 \sim N(12, 0.02)$$
$$X_2 \sim N(24, 0.03)$$
$$X_3 \sim N(18, 0.04)$$

If X_1, X_2 and X_3 are independent, determine $P(53.8 \le Y \le 54.2)$

(f) X_1, X_2, X_3 and X_4 are independent random variables. Let $Y_1=\ln X_1 \sim N(4,1)$, $Y_2=\ln X_2 \sim N(3,0.5)$, $Y_3=\ln X_3 \sim N(2,0.4)$, $Y_4=\ln X_4 \sim N(1,0.01)$.
For $W=e^{1.5}[X_1^{2.5}X_2^{0.2}X_3^{0.7}X_4^{3.1}]$, find $P(20{,}000 \le W \le 600{,}000)$

(g) The random variable X is the number of occurrences of an event over an interval of ten minutes. It can be assumed that the probability of an occurrence is the same in any two-time periods of an equal length. It is known that the mean number of occurrences in ten minutes is 5.3.

Which Probability Distribution does the random variable X satisfy?
Is it discrete or continuous?
What are the values of the Mean and Variance of X?
What is the probability that there are 8 occurrences in ten minutes?
What is the probability that there are less than 3 occurrences in ten minutes?

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