1705E650 Candidate's Seat No :

M.Sc. (Sem.-III) Examination
501 Statistics
May-2017

Time : 3 Hours] [Max. Marks : 70

1. (a) Exemplify the statement: The distribution specified under the null and alternative hypothesis not necessarily belong to the same family of the distributions for the application of NP lemma.

OR

(a) State and prove NP lemma for randomized test.

(b) Consider a population with three kinds of individuals labeled 1, 2 and 3. Suppose the proportion of individual of the three types are given by p(k, θ); k = ,2,3; where $0 < \theta < 1$ and

$$p(k,\theta) = \begin{cases} \theta^2, & \text{if k=1} \\ 2\theta(1-\theta), & \text{if k =2} \\ (1-\theta)^2, & \text{if k = 3} \end{cases}$$

Let $X_1, X_2, \ldots, X_n$ be a random sample from this population. Find the MP test for testing $H: \theta = \theta_0$ versus $K: \theta = \theta_1,\ 0 < \theta_0 < \theta_1 < 1$.

OR

(b) Let $X_1, X_2, \ldots, X_n$ be i.i.d. $N(\mu, 5)$ variates. Derive UMP test of size α to test $H: \mu \leq 0$ versus $K: \mu > 0$. Find also the power function of the test.

2. (a) Define MLR property. Let $X_1, X_2, \ldots, X_n$ be independent observations from exponential distribution with mean $\theta > 0$. Derive UMP test of size α using MLR property to test $H: \theta \geq \theta_0$ versus $K: \theta < \theta_0$.

OR

(a) Let ϕ be any unbiased test of level α for testing $H: \theta \in \Omega_H$ versus $K: \theta \in \Omega_K$. Suppose that the function $E(\phi(x))$, $\theta \in \Omega$ is continuous in θ then prove that ϕ is α- similar on boundary set Λ.

(b) Let $X_1, X_2, \ldots, X_n$ be a random sample from the distribution with p.d.f. $f(x,\theta) = e^{-(x-\theta)},\ x \geq \theta$. Derive UMP test of size α to test $H: \theta = \theta_0$ versus $K: \theta \neq \theta_0$. Hence find uniformly most accurate confidence interval for θ with confidence coefficient $1-\alpha$.

(b) Define test with Neyman structure. State and prove necessary and sufficient condition for all α- similar tests to have Neyman structure with respect to T, T be sufficient statistic involved in the exponential family of distributions.

3. (a) Describe SPRT. Derive stopping bounds of SPRT. Show that they are conservative at the boundary points.

OR

(a) State Wald's identity. Hence derive ASN and OC functions of SPRT.

(b) Consider the SPRT procedure as follows:

Continue the process if

$$-\left(\frac{n+1}{2}\right) < \sum_{i=1}^{n} X_i < \left(\frac{n+2}{2}\right),$$ where X_i's being the successive observations for testing

$H: P(X=-1) = P(X=1) = P(X=2) = 1/3$ versus $K: P(X=-1) = P(X=1) = 1/4;\ P(X=2) = 1/2$.

What is the probability that the procedure will terminate under K on or before second stage?

(P.T.O)

OR

(b) Derive SPRT for testing mean of normal distribution when variance is known. Obtain iys OC function.

4. (a) What is LRT? For large sample tests show that under LRT the distribution of $-2\log\lambda(X)$ is chi square With k d.f. for testing H: $\theta = \theta_0$ versus K: $\theta \neq \theta_0$, θ_0 is specified. What should be the value of k?

OR

(a) Obtain UMAU confidence interval for σ^2 of level $1-\alpha$ in case of $N(\mu, \sigma^2)$ distribution based on a random sample of size n. Also find expected length of this interval.

(b) Describe K-S test.

OR

(b) Describe Kruskal - Wallis test.

5. Answer the following:

i. State MP test.

ii. State unbiased test.

iii. Let $X \sim N(0, \sigma^2)$, and Y has exponential distribution with mean $2\sigma^2$ and X and Y are independent. We want to test H: $\sigma^2 \leq 1$ versus K: $\sigma^2 > 1$ at level α. Which of the following is true.

(A) UMP test does not exist (B) UMP test reject H; when $X^2 + Y$ is large
(C) UMP test is chi square test (D) UMP test reject H when $X^2 + Y$ is small

iv. Define level of significance.

v. State the approximate distribution of LRT test statistic to test H: $\theta_1 = \theta_2 = ... = \theta_0$ versus K: $\theta_1 \neq \theta_2 \neq ..... \neq .\theta k$.

vi. State the randomized test function.

vii. State UMP test.

viii. The K-S test procedures are based on

(A) vertical deviations between the observed and expected cumulative distribution functions.
(B) horizontal deviations between the observed and expected cumulative distribution functions.
(C) both horizontal and vertical deviations between the observed and expected cumulative distribution functions.
(D) None of the above

ix. To test the equality of two variances the appropriate non parametric test is

(A) chi square test. (B) F- test
(C) The Kruskal-Wallis test (D) The Siegel-Tukey test

x. State size of the randomized test.

xi. State TRUE or FALSE: UMPU test is always UMP test.

xii. State General formula of OC function of SPRT.

xiii. State general form of ASN function of SPRT.

xiv. State non parametric test in which linear rank statistic is used.

2005E760 Candidate's Seat No : ______

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Note: (i) Attempt all questions. (ii) All questions carry equal marks.

Q.1(a) What are the advantages and disadvantages of increased inventory? Briefly explain the objectives that must be fulfilled by an inventory control system.

OR

(a) Explain the various costs that are involved in inventory problems with examples.

(b) In usual notations show that for a probabilistic inventory model with instantaneous demand and no set-up cost, the optimum stock level Q can be obtained by the relationship

$$\sum_{d=0}^{Q-1} p(d) < \frac{C_2}{C_1 + C_2} < \sum_{d=0}^{Q} p(d).$$

OR

(b) Formulate and solve continuous probabilistic reorder point lot size model to determine optimal reorder point for a presented lot size. Lead time is finite. Shortages are allowed and fully backlogged.

Q.2 (a) What are the situations which makes the replacement of items necessary?

OR

(a) Suppose the cost of maintenance of a machine increases with time and its scrap value is constant. If time is measured in continuous units, then show that the average annual cost will be minimized by replacing the machine when the average cost to date becomes equal to the current maintenance cost.

(P.T.O)

(b) Explain group replacement concept and its applications.

OR

(b) State and prove mortality theorem as used for replacement problems.

Q.3 (a) How does PERT provide for uncertainty in activity time estimates? What is the rationale for using Beta probability distribution?

OR

(a) Briefly explain resource allocation with example in relation of network analysis.

(b) Discuss matrix solution method in network analysis.

OR

(b) Discuss time – cost trade – off procedure of a project.

Q. 4 (a) What is non-linear programming? Explain Lagrangian method for solving it.

OR

(a) Discuss Kuhn –Tucker necessary conditions for an optimal solution to a quadratic programming problem.

(b) Discuss simulation of queuing problems with examples.

OR

(b) Discuss simulation of maintenance problems with examples.

Q.5 Answer the following:

(i) One of the important reasons for carrying inventory is to
(a) improve customer service (b) get quantity discounts
(c) maintain operational capability (d) all of the above.

(ii) Operating decisions in an inventory system are concerned with
(a) order quantity (b) reorder level (c) customer service level (d) all of the above.

(iii) If the unit cost rises, then optimal order quantity
(a) increases (b) decreases (c) either increase or decrease (d) none of the above.

(iv) Define progressive failure.

(v) Define random failure.

(vi) The group replacement policy is suitable for identical low cost items which are likely to

(a) fail over a period of time (b) fail suddenly

(c) fail completely and suddenly (d) none of the above.

(vii) Define Free float.

(viii) Float or slack analysis is useful for

(a) projects behind the schedule only (b) projects ahead of the schedule only

(c) both (a) and (b) (d) none of the above.

(ix) If an activity has zero slack, it implies that

(a) it lies on the critical path (b) it is a dummy activity

(c) the project is progressing well (d) none of the above.

(x) Generally the PERT technique deals with the project of

(a) repetitive nature (b) non-repetitive nature

(c) deterministic nature (d) none of the above.

(xi) In Beale's method, we use the Kuhn-Tucker conditions.

(a) True (b) False.

(xii) Special simulation languages are useful because they

(a) reduce programme preparation time and cost
(b) have the capability to generate random variables
(c) require no prior programming knowledge
(d) all of the above.

(xiii) The important step required for simulation approach in solving a problem is to

(a) test and validate the model (b) design the experiment

(c) conduct the experiment (d) all of the above.

(xiv) An advantage of simulation as opposed to optimization is that

(a) several options of measure of performance can be examined
(b) complex real-life problems can be studied
(c) it is applicable in cases where there is an element of randomness in a system
(d) all of the above.

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