

M.Phil (Mathematics) Examination

Paper-I : Research Methodology

May-2017

[Max. Marks : 70]

Time : 3 Hours]

MAT601 Research Methodology and Problem Solving

Q.1 Attempt any three. 14

- (a) Prove that there is no real valued function on \mathbb{R} which is continuous at each rational but discontinuous at each irrational.
- (b) Define convex function. State and prove generalized AM-GM inequality using convex function theory.
- (c) Suppose $a_n > 0$, $s_n = a_1 + a_2 + \dots + a_n$ and $\sum a_n$ diverges. What can you say about the convergence of series $\sum \frac{a_n}{1+a_n}$ and $\sum \frac{a_n}{s_n}$? Explain.
- (d) Define (c, 1) and (c, 2) summability of the sequence (x_n) . Give an example of (c, 2) summable sequence that is not (c, 1) summable.
- (e) If $f : \mathbb{R} \rightarrow \mathbb{R}$ is a continuous, bounded function then show that $f(x) = x$ for some x .

Q.2 Let $A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$ be a matrix with real entries. 14

Find the eigenvalues of A . Find corresponding eigenvectors.

Find a matrix S so that $S^{-1}AS = D$, where D is a diagonal matrix.

Note that $A = SDS^{-1}$ and $A^k = SD^kS^{-1}$.

Verify that

$$\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} b \\ a+b \end{bmatrix}.$$

Suppose $\{F_k\}$ is a sequence such that

$$F_0 = 1, F_1 = 1, F_2 = 2, F_3 = 3, F_4 = 5, F_5 = 8, \dots, F_{k+1} = F_k + F_{k-1}, \dots$$

Prove by induction that

$$A^k \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} F_k \\ F_{k+1} \end{bmatrix}.$$

Find a formula for the Fibonacci number F_k .

Q.3 Attempt any four.

- (a) Give the names of at least three document class options.
- (b) Give the commands for the following mathematical notations:
- (i) \forall (ii) \neq (iii) \mathbb{R} .

- (c) Write down the input for the following output:
 $(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{n}b^n.$

- (d) Write down the input for obtaining the integral:

$$\int_{\frac{1}{2}}^3 x^2 \sin x dx$$

- (e) Explain how do we define or customize the command(with variable argument) to produce the vectors (x_1, x_2, \dots, x_n) , (y_1, y_2, \dots, y_n) or (z_1, z_2, \dots, z_n) which occur frequently in a certain document.
- (f) Write down the input for the following table:

a	b
c	d
e	f

Q.4 (a) Write down the outputs of any three of the following inputs:

3

- (i) `RotateLeft[{a, b, c, d, e, f}, -2]`
- (ii) `Split[{1, 2, 2, 3, 3, 3, 4, 5, , 2, 2, 2}]`
- (iii) `Plus[1. Power[x, 2], Power [Plus [y, z], 2]]`
- (iv) `FullForm[{a/b, 1/b^2, 2/b^2}]`
- (v) `TreeForm[x^3 + (1 + x)^2]`

- (b) Write a command for finding factors of the polynomial $1 + x^4$ over $\mathbb{Q}\sqrt{2}$. 2
- (c) Write a command for drawing a graph of $f(x) = 6x^3 - 5x^2 - 2x + 1$ for $-1 \leq x \leq 3/2$. 2
- (d) Write commands for all mathematical steps for finding the inverse of matrix 7

$$\begin{bmatrix} 3 & 0 & 2 \\ 2 & 0 & -2 \\ 0 & 1 & 1 \end{bmatrix}$$

- (a) What is the execution from the command "clc" and "clear"?
- (b) Which symbol is used for commenting statement?
- (c) What will be output for $A * B$ and $A . * B$ if
 $\gg A = [1\ 3\ 4; 6\ 4\ 3; 5\ 3\ 3];$
 $\gg B = [3\ 5\ 6; 8\ 7\ 6; 9\ 5\ 3];$
- (d) Give command to plots the given vectors on a vertical bar chart.
- (e) What is syntax for symbolic expansion of $(x + 1)^3$?
- (f) Give syntax to solve $x^2 + y^2 = 25$ and $x + y = 9$.
- (g) Give syntax to solve $\frac{d^2y}{dx^2} + 2y = x$.