Seat No. : _______________

TC-120

April-2013

M.Sc. (Sem.-II)

410 : Mathematics

(Partial Differential Equations)

Time : 3 Hours] **[Max. Marks : 70**

1. (a) Attempt any **one** : **7**

   (i) Find the general integral of $(x^2 + y^2)p + 2xyq = (x + y)z$.

   (ii) Verify that the Pfaffian differential equation

   $$yzdx + (x^2y - zx)dy + (x^2z - xy)dz = 0$$

   is integrable and find its integral.

   (b) Attempt any **two** : **4**

   (i) Find the general integral of $yzp + xzq = xy$

   (ii) Solve $yzdx + xzdy + xydz = 0$

   (iii) Form a partial differential equation by eliminating the arbitrary function F from $F(z - xy, x^2 + y^2) = 0$.

   (c) Answer very briefly : **3**

   (i) Form a partial differential equation by eliminating the parameters a and b from $2z = (ax + y)^2 + b$.

   (ii) Find the singular integral of $z - px - qy - p^2 - q^2 = 0$

   (iii) Find the envelope of $(x - a)^2 + (y - 2a)^2 + z^2 = 1$

2. (a) Attempt any **one** : **7**

   (i) Find a complete integral of the equation $z^2(p^2z^2 + q^2) = 1$.

   (ii) Find the integral surface of the differential equation

   $x(z + 2)p + (xz + 2yz + 2y)q = z(z + 1)$,

   passing through the curve $x_0 = s^2$, $y_0 = 0$ and $z_0 = 2s$.

   (b) Attempt any **two** : **4**

   (i) By Jacobi's method, solve the equation $u_x^2 + u_y^2 + u_z = 1$.

   (ii) Solve $z_x + z_y = z^2$ with the initial condition $z(x, 0) = f(x)$.

   (iii) Solve $zpq = p + q$

(c) Answer very briefly : **3**

(i) Write down the auxiliary equations of the p.d.e. $z^2 + zu_z - u_x^2 - u_y^2 = 0$.

(ii) Define admissible curve.

(iii) What is the complete integral of the equation $z = px + qy + p - q$?

3. (a) Attempt any **one** : **7**

(i) Find the solution of the equation $z = \frac{1}{2}(p^2 + q^2) + (p - x)(q - y)$, which passes through the x-axis.

(ii) Reduce the equation $u_{xx} - x^2 u_{yy} = 0$, to a canonical form.

(b) Attempt any **two** : **4**

(i) Of which type is the equation $x^{14}u_{xx} - 36u_{yy} + 7x^{13}u_x = 0$?

(ii) Find the characteristic strips of the equation $pq = z$ passing through the curve $C : x_0 = 0, y_0 = s, z_0 = s^2$.

(iii) Write down the d'Alembert's solution of one-dimensional wave equation.

(c) Answer very briefly : **3**

(i) Define second order semi-linear partial differential.

(ii) What is the canonical form of the semi-linear equation of hyperbolic type ?

(iii) Of which type is the Laplace's equation ?

4. (a) Attempt any **one** : **7**

(i) Solve the Dirichlet problem for the upper half plane.

(ii) Solve the Dirichlet problem for a rectangle.

(b) Attempt any **two** : **4**

(i) State (only) maximum and minimum principles.

(ii) Prove that the solution of Dirichlet problem, if it exists, is unique.

(iii) State Green's identities.

(c) Answer very briefly : **3**

(i) What is Dirichlet Problem ?

(ii) Give an example of a problem whose solution is not stable.

(iii) What are the Hadamard's conditions for a well posed problem ?

5. (a) Attempt any **one** : **7**

(i) Solve the heat conduction problem for an infinite rod.

(ii) Derive the necessary condition for the one parameter family of surfaces $f(x, y, z) = c$ to be a family of equipotential surfaces.

Show that the family of surfaces $x^2 + y^2 + z^2 = c$, $c > 0$ can form an equipotential family of surfaces, and find the general form of the potential function.

(b) Attempt any **two** : **4**

(i) Prove that the solution of the Neumann problem is unique up to the additional of a constant.

(ii) Let D be a bounded domain in $\mathbb{R}^2$, bounded by a smooth closed curve B. Let $\{u_n\}$ be a sequence of functions each of which continuous on $\overline{D} = D \cup B$ and harmonic in D. If $\{u_n\}$ converges uniformly on B, prove that $\{u_n\}$ converges uniformly on $\overline{D}$.

(iii) Write down Laplace equation in polar coordinates (r, θ).

(c) Answer very briefly. **3**

(i) State Neumann problem for the upper half plane.

(ii) What is the necessary condition for the existence of solution of interior Neumann problem ?

(iii) State Heat conduction problem for a finite rod.
