Seat No. : _____

AG-104

April-2016

M.Sc., Sem.-IV

510 : Mathematics

(Quantitative Techniques)

Time : 3 Hours] **[Max. Marks : 70**

1. (A) Attempt any **one** : **7**

   (i) The proportion of time per day that all checkout counters in a supermarket are busy follows a distribution

   $f(x) = kx^2 (1 - x)^9$ for $0 < x < 1$
   $= 0$ otherwise

   What is the value of the constant k so that $f(x)$ is a valid probability density function ?

   (ii) If the random variable X has a gamma distribution with parameters $\alpha = 1$ and $\lambda = 2$ then what is the probability density function of the random variable $Y = e^X$?

   (B) Attempt any **one** : **4**

   (i) Let $X_1, X_2, ..., X_n$ be a random variable, X_i having exponential distribution with parameter θ_i, i = 1, 2, ..., n. Then show that $Y - \min \{X_1, X_2, ..., X_n\}$ has exponential distribution with parameter $\sum_{i=1}^{n} \theta_i$.

   (ii) If X is a Poisson variate such that $P(X = 2) = 9P(X = 4) + 90 P(X = 6)$ find the mean of X and β_1, the coefficient of skewness.

   (C) Attempt any **one** : **3**

   (i) If X is uniformly distributed with mean 1 and variance 4/3 then find $P(X< 0)$.

   (ii) A man with n keys wants to open his door and tries the keys independently and at random. Find the probability to open the door if unsuccessful keys are not eliminated from further selection.

2. (A) Attempt any **one** : **7**

   (i) What will be the economic order quantity when ordering cost depends on purchase quantity ?

(ii) Discuss EOQ problem with investment constraint. Consider a shop which stores three items. The demand rate for each item is constant and can be assumed to be deterministic. Shortages are not allowed. The relevant data for the items is given in the following table :

Item :	**A**	**B**	**C**
Demand rate (units / year)	20	10	40
Holding cost (₹)	0.30	0.10	0.20
Set-up cost per lot (₹)	15	25	10
Purchase cost per unit (₹)	12	10	8

Determine the optimum lot size for each item when retailer has Rs. 1500.

(B) Attempt any **one** : **4**

(i) What do you mean by lead-time ?

(ii) Discuss the various costs involved in control of inventory.

(C) Attempt any **one** : **3**

(i) What is the nature of holding cost and ordering cost. Explain using graph.

(ii) Write algorithm to compute purchase quantity with one price break.

3. (A) Attempt any **one** : **7**

(i) Derive waiting time distribution of customers in queue.

(ii) Derive probability of n - customers in the system for ((M/M/c) : (∞/FCFS)) queue.

(B) Attempt any **one :** **4**

(i) A mathematics faculty guides student for a project with an exponential distribution having mean 20 minutes. If he discusses project in FCFS manner, and the arrival of students follows approximately Poisson with an average rate of 5 per 3-hour day, what is faculty's expected idle time each day ? How many jobs are ahead of the average students just enters in ?

(ii) Consider a single server queueing system with Poisson input, exponential service times. Suppose the mean arrival rate is 3 calling units per hour, the expected service time is 0.25 hours and maximum permissible number calling units in the system is two. Derive steady state probability distribution of the number of calling units in the system.

(C) Define any **Three** : **3**

(i) Traffic intensity

(ii) Jockeying

(iii) Kendall’s representation of queue system

(iv) Service in series

4. (A) Attempt any **one** : **7**

(i) Derive the replacement policy when the value of money change with time.

(ii) The cost of a machine is ₹ 70,000. The data found from experience is as follows :

Year	**1**	**2**	**3**	**4**	**5**
Resale value (₹)	52,000	35,000	22,000	15,000	9000
Cost of spares (₹)	5000	5750	6000	7800	8000
Cost of labor (₹)	24,000	26,000	28,000	32,000	37,000

When should the machine be replaced ?

(B) Attempt any **one** : **4**

(i) Machine A costs ₹ 3,600. Annual operating costs are ₹ 40 for the first year and then increases by ₹ 360 every year. Assume that machine A has no resale value, determine the best replacement age ?

(ii) For a diode, the following failure rates have been observed.

Week	1	2	3	4	5
% failing by end of week	10	20	45	75	100

There are 1000 diodes in use, and it costs ₹ 3 to replace an individual diode that has burnt out. If all the bulbs were replaced simultaneously, it would cost ₹ 0.25 per diode. It is proposed to replace all the diodes at fixed intervals, whether or not they have failed or not and to continue replacing failed diodes as they fail. When should all the diodes be replaced ?

(C) Choose the correct answer : **3**

(i) When time value of money is considered

a. costs need to be discounted,

b. timing of incurrence of costs is important,

c. the present value factors serve as the weignts.

d. all of the above.

(ii) Under group replacement policy

a. group as well as individual replacements are done.

b. all the items are replaced, irrespective of the fact that items have failed or have not failed.

c. the optimal group replacement interval is determined at the point where the sum of group replacement per unit of item and the cost of individual replacement is maximum.

d. none of the above.

(iii) Staff replacement policy

a. arises due to resignation, retirement, or death of a staff from time to time.

b. is like replacement policy for items whose efficiencies decrease gradually.

c. can be easily formulated because people retire at known time.

d. does not yield the optimum replacement interval.

5. (A) Attempt any **one** : **7**

(i) A retailer deals in perishable commodity. The daily demand and supply are variables. The data for the past 500 days are as follows.

Supply		**Demand**	
Availability (kg.)	**No. of days**	**Demand (kg.)**	**No. of days**
10	40	10	50
20	50	20	110
30	190	30	200
40	150	40	100
50	70	50	40

The retailer buys the commodity at ₹ 20 per kg. and sells it at ₹ 30 per kg. Any commodity remains at the end of the day, has no sale value. Moreover, the loss on any unsatisfied demand is ₹ 8 per kg Given the following random numbers, stimulate 6 days sales, demand and profit. **7**

(31,18) (63,84) (15,79) (07,32) (43,75) (81,27)

The first random number in the pair is for supply and the second random number is for demand.

(ii) The company produces 100 TVs. The daily production varies from 80 to 120 depending upon the availability of row materials and other facilities.

Production per day	80	85	90	95	100	105	110	115	120
Probability	0.04	0.09	0.12	0.14	0.11	0.10	0.20	0.12	0.08

The finish TVS are transported in a specially designed vehicle of capacity 100 TVs. Using following random numbers

80 81 76 75 64 43 18 26 10 12 65 68 69 61 57

Simulate the process to find out

(i) What will be the average number of TVs waiting in the company ?

(ii) What will be the average number of empty space in the vehicle ?

(B) Attempt any **one** : **4**

(i) Explain steps of Simulation.

(ii) Give an example to simulation with steps to be performed ?

(C) Attempt any **one :** **3**

(i) Outline the limitations of simulation.

(ii) When simulation should be used ?