Seat No. : _______________

AC-102

April-2016

B.Sc., Sem.-VI

CC-308 : Mathematics
(Analysis-II)

Time : 3 Hours] **[Max. Marks : 70**

Instructions : (1) **All** the **five** questions are compulsory.
(2) Each question is of **14** marks.

1. (a) Prove (i) Define : Riemann integrable function on [a, b]. Let a function $f(x) = \frac{x}{3}$ and $P = \left\{0, \frac{1}{n}, \frac{2}{n}, \frac{3}{n}, ... \frac{n}{n}\right\}$ be a partition of [0, 1] then compute $\lim_{n \to \infty} U_p(f)$ and $\lim_{n \to \infty} L_p(f)$.

OR

Let f be bounded on [a, b] then f is integrable on [a, b] if and only if for every $\varepsilon > 0$, there is a partition P of [a, b] such that $U_p(f) - L_p(f) < \varepsilon$.

(b) If $f \in R$ [a, b] and a < c < b then prove that f is R-integrable on [a, c] and on [c, b] and $\int_a^b f = \int_a^c f + \int_c^b f$

OR

State and prove fundamental theorem of calculus. Is the converse of the statement $f \in R[a, b] \Rightarrow |f| \in R[a, b]$ true ? Explain.

2. (a) Define alternating series. If (a_n) is a decreasing sequence of positive real numbers converging to zero then prove that $\sum_{n=1}^{\infty} (-1)^{n+1} a_n$ converges. Discuss the absolute convergence of the series $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{(\sqrt{n+5} - \sqrt{n})}{n}$

OR

If (a_n) is a decreasing sequence of positive real numbers and $\sum_{n=1}^{\infty} a_n$ converges then prove that $\lim_{n \to \infty} na_n = 0$ or $na_n \to 0$ as $n \to \infty$. Give suitable name to this result. If $\sum_{n=1}^{\infty} a_n^2$ is a convergent series of real numbers then show that $\sum_{n=1}^{\infty} \frac{a_n}{n}$ is also convergent.

(b) State and prove Cauchy condensation test for the convergence of the series and hence discuss the convergence of the series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ where, $p \in R$.

OR

If $\sum_{n=1}^{\infty} a_n$ converges and all $a_n > 0$ the show that

(i) $\sum a_n^2$

(ii) $\sum \frac{a_n}{1+a_n}$

(iii) $\sum \frac{a_n}{1-a_n}$; $a_n \neq 1$ are convergent.

3. (a) If the series $\sum_{n=0}^{\infty} b_n$ is a series of non-negative terms converges to X then prove that every rearrangement of the series $\sum_{n=0}^{\infty} b_n$ converges to the same sum X.

OR

Prove : If $\sum_{n=0}^{\infty} b_n x^n$ converges for some $x = c \neq 0$, then the series converges absolutely for all x with $|x| < |c|$

(b) Discuss the convergence of the following power series stating interval of convergence :

(i) $\sum_{n=1}^{\infty} \frac{7^n x^n}{(n-1)!}$

(ii) $\sum_{n=0}^{\infty} \frac{1}{n!} x^n$

(iii) $\sum_{n=0}^{\infty} (n+2)^n \frac{x^n}{(n+1)^n}$

OR

Prove : If $\int_a^{\infty} g(x)\, dx$ converges and $0 \leq f(x) \leq g(x)$ for all $x \in [a, \infty)$, then $\int_a^{\infty} f(x)\, dx$ converges and $\int_a^{\infty} f \leq \int_a^{\infty} g$. Test convergence : (i) $\int_0^1 \frac{1}{x(1-x)}\, dx$. (ii) $\int_0^{\infty} \frac{2x dx}{9 + x^2}$

4. (a) State Taylor's theorem. Using Lagrangian form for the remainder, for $-1 < x < 1$ show that $\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \dots\dots\dots + (-1)^{n-1}\frac{x^n}{n} + \dots\dots$

OR

For any real x, obtain Maclaurin series expansion of $\cos x$ hence, deduce the series for $\cos 0.2$.

(b) Obtain the power series solution of the differential equation $(1 - x)y' = -3y$ with the initial condition $y(0) = 2$.

OR

If $f(x) = (1 + x)^m$ then for $|x| < 1$, prove that

$$(1 + x)^m = 1 + mx + \frac{m(m-1)}{2!}x^2 + \dots\dots + \frac{m(m-1)\dots(m-n+1)}{n!}x^n + \dots\dots$$

5. Attempt any **seven** :

(i) Show that $\int_{-1}^{1} |x|\, dx = 1$.

(ii) Let $f(x) = \frac{x^2}{4}$, $n \in N$, $P = \left\{0, \frac{1}{4}, \frac{2}{4}, \frac{3}{4}, 1\right\}$ be a partition of [0, 1]. Is f R-integrable on [0, 1] ? Justify.

(iii) Discuss the conditional convergence of $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\left(\sqrt{n^2+1} + n\right)}$

(iv) Find the radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{1}{n}x^n$.

(v) Find the Cauchy product of $\sum_{n=0}^{\infty} \frac{(-1)^n}{\sqrt{n}}$ with itself.

(vi) Test the convergence of $\int_{-\infty}^{0} 2^x dx$.

(vii) State the Taylor series for f about x_0.

(viii) Examine the validity of the statement $e = 1 + 1 + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \dots\dots\dots$

(ix) If $y(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + \dots\dots\dots$ and $-3y(x) = y'(x)$ then obtain relation among the coefficients.
