Seat No. : _______________

NE-102

December-2015

B.Sc., Sem.-V

Core Course-304 : Mathematics

Time : 3 Hours] **[Max. Marks : 70**

Instructions : (1) There are **5** questions.

(2) **All** questions are compulsory.

1. (a) Let $S \subset \mathbb{R}^n$, prove that S is convex set if and only if every finite convex linear combination of element in S is also in S. **(7)**

OR

Define convex hull. Prove that convex hull is always convex.

(b) Prove that $S_k = \{\bar{x} \in R^n : \|\bar{x}\| \leq k\}$, is a convex set in R^n, where k is positive real number. Is the set $S_1 = \{\bar{x} \in E^n : \|\bar{x}\| = k\}$ is convex set ? Justify. **(7)**

OR

Mr. Modi required atleast 10, 12 and 12 units of chemicals A, B and C for his garden. One jar of liquid product contains 3, 2 and 1 unit of A, B, C respectively. A dry product contains 1, 2 and 4 units of A, B, C per carton. If the liquid product sells for ₹ 3 per jar and the dry product sells for ₹ 2 per carton, pose a LPP to show how may of each should he purchase to minimize the cost and to meet the requirements and solve using graphical method.

2. (a) If S_F is a non-empty bounded subset of $\mathbb{R}^n$ for a given L.P. problem then prove that the optimum solution exists at one of the vertex of S_F, where S_F = The set of all feasible solution of L.P. problem. **(7)**

OR

Prove that every vertex of the set of all feasible solution, is a basic feasible solution of an L.P. problem.

(b) Solve the following L.P.P. by simplex method : **(7)**

Maximize $Z = 3x_1 + x_2 + 3x_3$

subject to condition
$$2x_1 + x_2 + x_3 \leq 2$$
$$x_1 + 2x_2 + 3x_3 \leq 5$$
$$2x_1 + 2x_2 + x_3 \leq 6$$
with $x_1, x_2, x_3 \geq 0$.

OR

Apply the two phase process to solve that the following L.P problem :

$$\text{Max } Z = 2x_1 - x_2 + x_3$$

subject to condition

$$x_1 + x_2 - 3x_3 \le 8$$
$$4x_1 - x_2 + x_3 \ge 2$$
$$2x_1 + 3x_2 - x_3 \ge 4$$
$$\text{with } x_1, x_2, x_3 \ge 0.$$

3. (a) Define Dual of L.P.P. Explain about simplex multipliers. **(7)**

OR

State and prove the Fundamental theorem of Duality.

(b) Solve the following L.P.P. using simple method : **(7)**

$$\text{Min } Z = 3x_1 + 5x_2 + 2x_3$$

Subject to condition

$$-x_1 + 2x_2 + 2x_3 \ge 3$$
$$x_1 + 2x_2 + x_3 \ge 2$$
$$-2x_1 - x_2 + 2x_3 \ge -4$$
$$\text{with } x_1, x_2 \ge 0.$$

OR

Find the optimal of the following problem by solving its dual :

$$\text{Maximize } Z = 3x_1 + 4x_2$$

subject to condition

$$x_1 + x_2 \le 10$$
$$2x_1 + 3x_2 \le 18$$
$$x_1 \le 8$$
$$x_2 \le 6$$
$$\text{with } x_1, x_2 \ge 0.$$

4. (a) Prove that for m origins to n destinations balance type transportation problem has m + n – 1 basic variables. **(7)**

OR

Prove that necessary and sufficient condition for the existence of a feasible solution to a transportation problem is $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$.

(b) Solve following transportation problem for minimum transportation cost MODI's Method : **(7)**

From ↓ To →	**A**	**B**	**C**	**D**	$\mathbf{a_i}$
I	42	48	38	37	160
II	40	49	52	51	150
III	39	38	40	43	190
$\mathbf{b_j}$	80	90	110	160	

OR

In following assignment problem, 5 different jobs are to be assigned to 5 different operators such that the total processing time is minimized. The following matrix represent processing time in hours :

Salesman ↓	**Sales districts**				
	A	**B**	**C**	**D**	**E**
1	32	38	40	28	40
2	40	24	28	21	36
3	41	27	33	30	37
4	22	38	41	36	36
5	29	33	40	35	39

Solve this assignment problem.

5. Answer the following multiple choice : **(14)**

(a) Let $S_1 = \{((x_1, x_2) : 2x_1 + 3x_2 = 5\}$, $S_2 = \{(1, 1)\}$ be two subset of $\mathbb{R}^2$. Then $S_1 \cap S_2$ is

(i) a convex
(ii) not a convex set
(iii) cannot say
(iv) none of these.

(b) Consider the set $S = \{(x_1, x_2) : x_2^2 \leq x_1\}$. Then S has

(i) no vertex
(ii) finite number of vertex
(iii) infinite number of vertex
(iv) none of these

(c) Which of the following is true ?

(i) An assignment problem can be solved by simplex method.
(ii) An assignment problem can be solved by transportation method.
(iii) An assignment problem can be solved by Hungarian method.
(iv) All of the above

(d) A basic feasible solution is called ____________ if the value of atleast one basic variable is zero.

(i) Degenerate
(ii) Non-degenerate
(iii) Optimum
(iv) None of these

(e) Constraints in an L.P. model represents

(i) Limitation
(ii) Requirements
(iii) Balancing
(iv) All of the above

(f) If a primal LP problem has finite solution, then the dual LP problem should have

(i) Finite solution
(ii) Infeasible solution
(iii) Unbounded solution
(iv) None of these

(g) When the total supply is not equal to total demand in a transportation problem, then it is called
(i) Balanced
(ii) Unbalanced
(iii) Degenerate
(iv) None of these

(h) The solution to a transportation problem with m-rows and n-columns is feasible if number of positive allocations are
(i) m + n
(ii) m × n
(iii) m + n – 1
(iv) m + n + 1

(i) The solution space (region) of dual LP problem is unbounded due to
(i) an incorrect formulation of the LP model
(ii) objective function is unbounded
(iii) neither (i) nor (ii)
(iv) both (i) and (ii)

(j) If there are n workers and n jobs there would be
(i) n! solutions
(ii) (n –1)! solutions
(iii) (n!)n solutions
(iv) n solutions

(k) The system of equations $x_1 + x_2 + x_3 = 4$, $2x_1 + x_2 - 5x_3 = 3$ is equivalent to the following system with inequalities ?
(i) $x_1 - x_2 + x_3 \le 4$, $2x_1 + x_2 - 5x_3 \le 3$, $-x_1 + 2x_2 + 6x_3 \ge 7$
(ii) $x_1 - x_2 + x_3 \le 4$, $2x_1 + x_2 - 5x_3 \le 3$, $x_1 + 2x_2 + 6x_3 \le 1$
(iii) $x_1 - x_2 + x_3 \le 4$, $2x_1 + x_2 - 5x_3 \le 3$, $2x_1 - 4x_3 \le 1$
(iv) $x_1 - x_2 + x_3 \le 4$, $2x_1 + x_2 - 5x_3 \le 3$, $3x_1 - 4x_3 \ge 7$

(l) In phase I of the two phase method an artificial variable turns out to be at positive level in the optimal table of Phase-I, then the LPP has
(i) no feasible solution
(ii) unbounded solution
(iii) optimal solution
(iv) none of these

(m) Consider the set S = $\{(x_1, x_2) : x_1 + x_2 \ge -1, x_1 \le 0, x_2 \le 2\}$. Then S has
(i) no vertex
(ii) infinite number of vertices
(iii) only two vertices
(iv) none of these.

(n) The dual of dual LP problem is
(i) primal
(ii) dual
(iii) cannot say
(iv) none of these.