Seat No. : _____

AA-118

April-2016

T.Y., MBA Integrated

Operations Research

Time : 3 Hours] **[Max. Marks : 100**

Instructions : (1) Graph papers shall be provided on demand.

(2) Non-programmable scientific calculators are permitted.

1. Attempt any **two** : **20**

(a) Discuss the assumptions of linear programming problem.

(b) A manufacturer produces three products A, B and C. Each product requires processing on two machines M_1 and M_2. The time required to produce one unit of each product on a machine is given in the table below :

Product	**Time to produce one unit (hours)**	
	Machine M_1	**Machine M_2**
A	0.5	0.6
B	0.7	0.8
C	0.9	1.05

There are 85 hours available on each machine; the operating cost is ₹ 5/hour for machine M_1 and ₹ 4/hour for Machine M_2. The product requirements are at least 90 units of A, at most 80 units of B and at least 60 units of C. The manufacturer wishes to meet the requirements at minimum cost. Formulate the problem as linear programming problem.

(c) Solve the following problem by graphical method :

Max $Z = ₹ [-0.1x_1 + 0.5x_2]$

Subject to constraints

$$2x_1 + 5x_2 \leq 80$$

$$x_1 + x_2 \leq 20$$

$$-0.1x_1 + 0.5x_2 \geq 6$$

$$x_1, x_2 \geq 0$$

2. Food X contains 6 units of A per gram and 7 units of B per gram and costs 12 paise per gram. Food Y contains 8 units of vitamin A per gram and 12 units of vitamin B per gram and costs 20 paise per gram. The daily minimum requirement of vitamin A and vitamin B is 100 unit and 120 units respectively. Find the minimum cost of product mix by the simplex method. **20**

OR

Use dual simplex method to solve the following linear programming problem.

Max $Z = -3x_1 - 2x_2$

Subject to constraints

$x_1 + x_2 \geq 1$

$x_1 + x_2 \leq 7$

$x_1 + 2x_2 \geq 10$

$x_2 \leq 3$

$x_1, x_2 \geq 0$

3. (a) Obtain the dual of the following primal problem.

Minimize $Z = 3x_1 - 2x_2 + x_3$

Subject to constraints

$2x_1 - 3x_2 + x_3 \leq 5$

$4x_1 - 2x_2 \geq 9$

$-8x_1 + 4x_2 + 3x_3 = 8$

$x_1, x_2 \geq 0$, x_3 is unrestricted **10**

(b) A company manufactures three products A, B and C using three types of inputs X, Y and Z in different proportions. The following matrix gives requirements of various inputs (in kg.) per unit product (one kg) of the three products.

Products	**Inputs**		
	X	Y	Z
A	4	8	8
B	4	6	4
C	8	4	0

The per unit profits for the three products are ₹ 20, ₹ 40 and ₹ 10 respectively. The company has 800 kg of input X, 1800 kg of input Y and 500 kg of input Z. The following final simplex table gives the optimal solution. **10**

C_B	Basis	x_1	x_2	x_3	S_1	S_2	S_3	b_i
10	x_3	$-\frac{1}{2}$	0	1	$\frac{1}{8}$	0	$-\frac{1}{8}$	$\frac{75}{2}$
0	S_2	–2	0	0	$-\frac{1}{2}$	1	–1	900
40	x_2	2	1	0	0	0	$\frac{1}{4}$	125
	C_j	20	40	10	0	0	0	
	$\Delta_j = C_j - z_j$	–55	0	0	$-\frac{5}{4}$	0	$-\frac{35}{4}$	

Determine :

(i) What would be the effect on the current optimal solution if the amount of input Z is increased to 700 kg.

(ii) What is the range for the contribution of product B over which the present product mix remains optimal.

(iii) A new product D is proposed to be added. The input requirements for it are 3 of X, 4 of Y and 6 of Z and the unit profit is ₹ 25. Is it worthwhile adding this product ?

4. Attempt any **two** : **20**

(a) A dairy firm has three plants located in a state. Daily milk production at each plant is as follows :

Plant	**Unit (million litres)**
1	6
2	1
3	10

Each day the firm must fulfill the needs of its four distribution centres. Milk requirement at each centre is as follows :

Distribution Centre	**Unit (million litres)**
1	7
2	5
3	3
4	2

Cost of shipping one million litres of milk from each plant to each distribution centre is given in the following table in hundreds of rupees :

Distribution Centres

		1	2	3	4
Plants	1	2	3	11	7
	2	1	0	6	1
	3	5	8	15	9

Formulate the linear programming mathematical model for the problem. (Do not solve the problem).

(b) A manufacturer wants to ship 22 loads of his product as shown below. The matrix gives the kilometres from sources of supply to the destinations.

Destination

		D_1	D_2	D_3	D_4	D_5	**Supply**
Source	S_1	5	8	6	6	3	8
	S_2	4	7	7	6	5	5
	S_3	8	4	6	6	4	9
Demand		4	4	5	4	8	

Shipping cost is ₹ 10 per load per kg. What shipping schedule should be used to minimize total transportation cost ?

(c) The following table represents the supply from the plants, the requirement at the distribution centres and the unit transportation costs.

Distribution Centres

		A	B	C	**Supply**
Plants	1	11	13	25	150
	2	13	15	35	300
Requirement		150	150	150	

When each plant is also considered a destination and each distribution centre is also considered as origin, the additional cost data are as follows :

To Plant

		A	B
From plant	A	0	75
	B	11	0

To Distribution Centre

		A	B	C
From Distribution Centre	A	0	33	11
	B	11	0	13
	C	75	13	0

To Plant

		1	2
From Distribution Centre	A	13	25
	B	35	13
	C	55	65

Find the optimal shipping schedule for the trans-shipment problem.

5. (a) The captain of a cricket team has to allot five middle batting positions to five batsmen. The average runs scored by each batsman at these positions are as follows :

Batsman	**Batting Position**				
	I	II	III	IV	V
P	40	40	35	25	50
Q	42	30	16	25	27
R	50	48	40	60	50
S	20	19	20	18	25
T	58	60	59	55	53

Find the assignment of batsmen to positions which would give the maximum number of runs. **10**

(b) A salesman wants to visit cities A, B, C, D and E. He does not want to visit any city twice before completing his tour of all the cities and wishes to return to the point of starting journey. Cost of going from one city to another (in rupees) is shown in the table. Find the least cost route. **10**

	A	**B**	**C**	**D**	**E**
A	∞	2	5	7	1
B	6	∞	3	8	2
C	8	7	∞	4	7
D	12	4	6	∞	5
E	1	3	2	8	∞

OR

An airline, that operates seven days a week, has a time table shown below. Crews must have a minimum layover of 6 hours between flights. Obtain the pairing of flights that minimizes layover time away from home. For any given pairing, the crew will be based at the city that results in the smaller layover.

Flight	**Delhi Depart**	**Calcutta Arrive**	**Flight**	**Calcutta Depart**	**Delhi Arrive**
1	7 : 00 a.m.	9 : 00 a.m.	101	9 : 00 a.m.	11 : 00 a.m.
2	9 : 00 a.m.	11 : 00 a.m.	102	10 : 00 a.m.	12 : 00 noon
3	1 : 30 p.m.	3 : 30 p.m.	103	3 : 30 p.m.	5 : 30 p.m.
4	7 : 30 p.m.	9 : 30 p.m.	104	8 : 00 p.m.	10 : 00 p.m.

For each pair also, mention the town where the crew should be based. **20**