Seat No. : _______________

NC-102

December-2015

B.Sc., Sem.-V

Core Course-302 : Mathematics
(Analysis – I)

Time : 3 Hours] **[Max. Marks : 70**

Instructions : (1) **All** questions are compulsory.
(2) Figure to right indicate full marks of the questions/sub question.
(3) Notations used in this question paper carry their usual meaning.

1. (a) Prove that countable union of countable sets is again countable. **7**

OR

Let $f : A \to B$ and let G and H be subsets of B. Show that $f^{-1}(G \cap H) = f^{-1}(G) \cap f^{-1}(H)$.

(b) State and prove (rational) density theorem. **7**

OR

Prove that an upper bound "α" of a non empty set S is the supremum of S if and only if for every $\in > 0$ there exists an element $S_\in$ in S such that $\alpha - \in < S_\in$.

2. (a) State and prove squeez theorem for sequences. **7**

OR

If $\{x_n\}$ is a Cauchy sequence of real numbers then prove that $\{x_n\}$ is convergent.

(b) State and derive Nested interval theorem. **7**

OR

Prove that a bounded sequence monotonic sequence increases.

3. (a) Prove that if $\lim\limits_{x \to a} f(x)$ exists; it is unique. **7**

OR

If $\lim\limits_{x \to a} f(x) = L$ and $\lim\limits_{x \to a} g(x) = M \neq 0$, then prove that $\lim\limits_{x \to a} \dfrac{f(x)}{g(x)} = \dfrac{L}{M}$. **7**

(b) If function f is continuous at "a" and function g is continuous at "f(a)", then gof is continuous at "a". **7**

OR

If function f is continuous at "a" and $\{x_n\}$ is a sequence converging to "a'. Then prove that the sequence $\{f(x_n)$ is convergent to f(a).

4. (a) State and prove mean value theorem. **7**

OR

State and derive Darboux's theorem.

(b) If function f is differentiable at "x", then prove that function f is continuous at "x" also. Does converse hold ? Justify your answer. **7**

OR

Suppose that function f is continuous and one-to-one on [a, b] and is differentiable with $f^1(x_0) \neq 0$. Then prove that f^{-1} is also differentiable at $y_0 = f^1(x_0)$ and

$$(f^{-1})^{l}(y_0) = \frac{1}{(f^{l}f^{-1}(y_0))}.$$

5. Answer the following in brief : **14**

(a) In usual notations prove that [0, 1] ~ (0, 1).

(b) Give an example of a bounded sequence which is neither a Cauchy sequence nor convergent.

(c) If $f(0) = f^1(0) = 1$, then evaluate $\lim_{x \to 0} \frac{f(x) - 1}{x}$.

(d) Give an example of a continuous function which is not uniformly continuous.

(e) State only L'Hospital's Rule.

(f) Prove that every convergent sequence is Cauchy too.

(g) Evaluate : $\lim_{x \to 0} \frac{(1 + x)^{1/x} - e}{x}$.