Seat No. : _______________

NB-143

December-2015

S.Y. M.Sc. (CA & IT)

Discrete Mathematics

Time : 3 Hours] **[Max. Marks : 100**

1. Attempt any **four :** **20**

   (1) Construct a truth table for the proposition $(q \wedge p) \rightarrow (p \wedge r)$.

   (2) Show the following equivalence :

   $\sim(p \leftrightarrow q) \equiv (p \vee q) \wedge \sim (p \wedge q)$

   (3) Obtain the PCNF of $(p \rightarrow q) \rightarrow r$.

   (4) Show that $r \wedge (p \vee q)$ is a valid conclusion from the premises

   $p \vee q$, $q \rightarrow r$, $p \rightarrow m$ and $\sim m$.

   (5) Show that $\{\sim, \wedge\}$ is a functionally complete set.

2. Attempt **all** : **20**

   (1) Find the domain, range and inverse of the relation R = {(a, b)/a divides b} defined on A = {1, 2, 3, 4, 5, 6}

   (2) Let R = {(1, 2), (2, 3), (2, 2), (3, 2), (2, 1), (1, 1), (2, 4), (3, 4), (4, 1)} be a relation on a set X = {1, 2, 3, 4}. Draw the graph and find the matrix of R.

   (3) Let A = {1, 3, 5, 7} and B = {2, 3, 4, 5}. Find the following

   (i) $A \cup B$, (ii) $A \cap B$, (iii) A – B and (iv) B – A.

   (4) Define :

   (i) Statement function.

   (ii) Free variable

   (iii) Bound variable

   (iv) Scope of quantifier

   (v) Universe of discourse

3. Attempt **all** : **20**

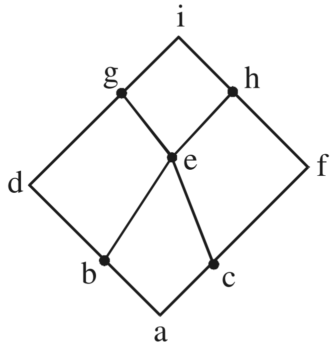
(1) Let A = {1, 2, 3, 4} and R = {(1, 1), (1, 2), (2, 3), (3, 2), (4, 3), (3, 4)}. Find the transitive closure of R.

(2) Define a lattice. Express the poset ({2, 3, 6, 12}, /) with the help of a Hasse diagram. Is this poset a lattice ? Explain.

(3) Define complement of an element in a lattice and a complemented lattice. Find complements of every element of the lattice $(D_{75}, 1)$.

$(D_{75}$ = set of all factors of 75)

(4) Find the greatest, least, minimal and maximal elements of the poset.



Also find the upper bounds, lower bounds, glb and lub of the sets.

(i) {g, f} and (ii) {d, e, f}

4. Attempt **all :** **20**

(1) Simplify the Boolean Expressions :

(i) $(a * b)' \oplus (a \oplus b)'$

(ii) $(a * c) \oplus c \oplus [(b \oplus b') \oplus c]$

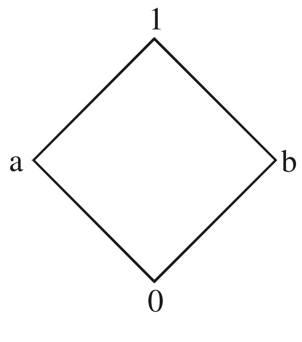
(2) Obtain the product of sums canonical form in three variables x_1, x_2 and x_3 of the expression $x_1 * x_2$.

(3) Show that in a Boolean Algebra,

$a \le b \Leftrightarrow a' \oplus b = 1 \Leftrightarrow a * b' = 0$

(4) Find the value of the Boolean Expression

$(x_1 * x_2 * x_3) \oplus (x_1' * x_2 * x_3) \oplus (x_1 * x_2' * x_3) \oplus (x_1' * x_2' * x_3')$ for the Boolean Algebra.



for $x_1 = a$, $x_2 = b$, $x_3 = 1$

5. Attempt any **four :** **20**

(1) Show that the set of all integers $\mathbb{Z}$ forms an abelian group with respect to the operation $*$ defined as $a * b = a + b + 1 \ \forall\, a, b \in \mathbb{Z}$.

(2) If $a^2 = e$ for all $a \in G$ in a group $(G, *)$, then prove that G is abelian.

(3) Define a cyclic group. Is $(\mathbb{Z}, +)$ a cyclic group ? Explain.

(4) If $(G, *)$ is a group and a, b, c are elements of G, then, prove that

(i) $a * b = a * c \Rightarrow b = c$

(ii) $b * a = c * a \Rightarrow b = c$.

(5) (i) Investigate whether the following permutation is odd or even :

$$F = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 7 & 3 & 1 & 8 & 5 & 6 & 2 & 4 \end{pmatrix}$$

(ii) If $g = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 1 & 5 & 4 & 6 & 2 \end{pmatrix}$, find g^2 and g^{-1}.