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## (Quantum Mechanics-1 \& Mathematical Physics)

Time : 3 Hours]
[Max. Marks : 70

1. (a) Find out minimum energy of He -atom using variation method.

## OR

Explain : variation method. Show that $\left(\mathrm{W}-\mathrm{E}_{0}\right) \leq\left[\left\langle\mathrm{H}^{2}\right\rangle_{\psi}-\mathrm{W}^{2}\right]^{1 / 2}$.
(b) Discuss Stark effect for the first excited state of hydrogen atom. Obtain eigen values and eigen vectors. Explain how degeneracy is not completely removed.
Given : $\left|\mathrm{U}_{200}\right\rangle=\left[\frac{1}{32 \pi \mathrm{a}^{3}}\right]^{1 / 2}\left[2-\frac{\mathrm{r}}{\mathrm{a}}\right] \exp \left(-\frac{\mathrm{r}}{2 \mathrm{a}}\right)$ and

$$
\left|\mathrm{U}_{210}\right\rangle=\left[\frac{1}{32 \pi \mathrm{a}^{3}}\right]^{1 / 2}\left[\frac{\mathrm{r}}{\mathrm{a}}\right] \exp \left(-\frac{\mathrm{r}}{2 \mathrm{a}}\right) \cos \theta .
$$

OR
Set-up Hamiltonian for Hydrogen molecule. Solving Schrodinger equation, obtain energies of symmetric and anti-symmetric states.
2. (a) What is propagator ? Write differential equation for propagator and obtain propagator for free particle.

## OR

Obtain Bohr-Sommerfeld quantization condition and find energy of simple harmonic oscillator.
(b) Discuss sudden approximation and obtain expression for transition probability.

## OR

Obtain solution of the time dependent Schrodinger equation. What do you mean by retarded propagator? Obtain equation for propagator.
3. (a) Obtain Laplace transform of $\mathrm{f}(\mathrm{t})=\mathrm{t}^{\mathrm{n}} ; \mathrm{t}>0 ; \mathrm{n}>-1$

## OR

Obtain Laplace transform of
(i) $\mathrm{f}(\mathrm{t})=\mathrm{t} \sin \mathrm{at} ; \mathrm{t}>0 ; \mathrm{a}=$ constant.
(ii) $\mathrm{f}(\mathrm{t})=\mathrm{t} \cos \mathrm{at} ; \mathrm{t}>0 ; \mathrm{a}=\mathrm{constant}$
(b) Describe the method of solving the differential equation by Laplace transform.

## OR

A particle of mass 3 g moves on the x -axis and is attracted towards a fixed point with a force whose numerical value is 12 x . Assuming that the particle is initially at rest at $x=5$, determine the position of particle at any time $t$,
(i) when there is no other force.
(ii) when there is a damping force whose numerical value is 6 times the instantaneous velocity.
4. (a) Define a class and subgroup. Write four postulates of subgroup. Discuss two interesting results involving classes.

What do you understand by 'closer property of the group' ? Define left and right coset and discuss meaning of disjoint set.
(b) Show that:
(i) $\mathrm{A}_{\mathrm{ik}}+\mathrm{B}_{\text {ik }}=\mathrm{C}_{\mathrm{ik}}$
(ii) $\mathrm{A}_{\mathrm{ik}}-\mathrm{B}_{\mathrm{ik}}=\mathrm{D}_{\mathrm{ik}}$

Where $\mathrm{A}_{\mathrm{ik}}, \mathrm{B}_{\mathrm{ik}}, \mathrm{C}_{\mathrm{ik}}$ and $\mathrm{D}_{\mathrm{ik}}$ are Tensors of same rank.

## OR

Define a 'tensor' and show that,

$$
\mathrm{V}^{2}=\mathrm{V} . \mathrm{V}=\Sigma_{\mathrm{i}} \Sigma_{\mathrm{j}} \mathrm{~g}_{\mathrm{ij}} \mathrm{~V}_{\mathrm{i}} \mathrm{~V}_{\mathrm{j}}
$$

5. Answer the following questions :
(1) Define Heaviside unit function.
(2) What is exchange integral?
(3) In WKB method expansion of wave function is in power series of $\qquad$ .
(4) Show that $\Delta_{I, I I}=\Delta_{I I, I}$.
(5) Write normalized eigen-states for the energy $\mathrm{W}^{(1)}= \pm 3 \mathrm{eEa}$.
(6) If wave function for free particle is $\psi=\mathrm{e}^{-\alpha \mathrm{r}}$ with $\alpha=$ constant, find $\mathrm{w} \equiv\langle\mathrm{H}\rangle_{\psi}$.
(7) What will be perturbed Hamiltonian when Helium-atom is placed in the uniform electric field of intensity E ?
(8) If $\mathrm{S}_{\mathrm{ij}}$ is a symmetric tensor and $\mathrm{A}_{\mathrm{ij}}$ is an anti-symmetric tensor, what is product of $\mathrm{S}_{\mathrm{ij}}, \mathrm{A}_{\mathrm{ij}}$ ?
(a) a tensor of mixed symmetry
(b) an anti-symmetric tensor
(c) a symmetric tensor
(d) zero
(9) If $A^{\mu}$ and $B_{v}$ are components of contravariant and covariant tensors, what is the nature of the quantity $\mathrm{A}^{\mu} \mathrm{B}_{\mathrm{v}}$ ?
(a) zero
(b) an invariant
(c) a covariant
(d) a mixed tensor of rank 2
(10) What is 'quotient group'?
(11) Show that A (B.C) $=($ A.B) $C$
(12) Which one of the following statement is true for Laplace transformation?
(a) $L\left(y^{n}\right)=p^{n} L(y)-p^{n-1} y_{0}-p^{n-2} y_{0}^{\prime}-p^{n-3} y_{0}^{\prime \prime}-\ldots . .-y_{0}^{n-1}$
(b) $L\left(y^{n}\right)=p^{n} L(y)-p^{n-1} y_{0}-p^{n-2} y_{0}^{\prime}-p^{n-3} y_{0}^{\prime \prime}-\ldots . .-y_{0}^{n}$
(c) $\quad \mathrm{L}\left(\mathrm{y}^{\mathrm{n}}\right)=\mathrm{p}^{\mathrm{n}} \mathrm{L}(\mathrm{y})-\mathrm{p}^{\mathrm{n}-1} \mathrm{y}_{0}+\mathrm{p}^{\mathrm{n}-2} \mathrm{y}_{0}^{\prime}-\mathrm{p}^{\mathrm{n}-3} \mathrm{y}_{0}^{\prime \prime}+\ldots . .+\mathrm{y}_{0}^{\mathrm{n}-1}$
(d) $\quad L\left(y^{n}\right)=p^{n} L(y)-p^{n-1} y_{0}-p^{n-2} y_{0}^{\prime}-p^{n-3} y_{0}^{\prime \prime}-\ldots . .-y_{0}^{n+1}$
(13) $\mathrm{L}\left(\mathrm{e}^{-\mathrm{at}}\right)=$ $\qquad$ $\operatorname{Re}(p+a)>0$.
(14) $\qquad$ is a kernel for Fourier transform.
