Seat No. : _______________

NG2-111

December-2015

M.Sc., Sem.-III

504 : MATHEMATICS

Mathematical Programming

Time : 3 Hours] **[Max. Marks : 70**

1. (a) Attempt any **one** : **7**

(i) Solve the following LPP using Big - M method :

Maximize $z = x_1 + 3x_2 + x_3$

subject to :

$$2x_1 + 3x_2 - x_3 = 5$$

$$x_1 + 5x_2 + 2x_3 = 12$$

and $x_1, x_2, x_3 \geq 0.$

(ii) Write the dual of the following LPP :

Maximize $P = 2x - 3y + 4z$

subject to:

$$2x + y + 3z \leq 7$$

$$x + y + z \geq 3$$

$$3x + 5y + 2z = 9$$

and $x, y, z \geq 0.$

From the solution of the dual, determine the solution of the given primal.

(b) Attempt any **one** : **4**

(i) What are the properties of Linear Programming Problem ?

(ii) A manufacturer produces two types of models M and N. Each M model requires 4 hours of grinding and 2 hours of polishing; whereas each N model requires 2 hours of grinding and 5 hours of polishing. The manufacturer has 2 grinders and 3 polishers. Each grinder works for 40 hours a week and each polisher works for 60 hours a week. Profit on model M is ₹ 3 and model N is ₹ 4. Whatever is produced in a week is sold in the market. How should the manufacturer allocate his production capacity to the two types of models so that he may make the maximum profit in a week ?

(c) Choose the correct **one** : **3**

(i) Linear programming is

(a) constrained optimization model;

(b) constrained decision - making model;

(c) mathematical programming model;

(d) all of the above.

(ii) Infeasibility is observed

(a) in computing the entering variable;

(b) in computing the leaving variable;

(c) artificial variable remains in the basis;

(d) none of the above.

(iii) The primal is a maximization model in m – equality constraints and n – non-negative variables. The dual has

(a) n constraints and m non-negative variables;

(b) is a minimization model;

(c) both (a) and (b);

(d) neither (a) nor (b).

2. (a) Attempt any **one** : **7**

(i) Find the optimum integer solution to the LPP

Maximize $Z = 2x_1 + 3x_2$

Subject to the constraints :

$$x_1 + 2x_2 \leq 4;$$

$$x_1 + x_2 \leq 3; x_1, x_2 \geq 0$$

(ii) Use Gomory's cutting plane method to solve the LPP

Maximize $Z = 3x_1 + 12x_2$

Subject to the constraints :

$$2x_1 + 4x_2 \leq 7$$

$$5x_1 + 3x_2 \leq 15; x_1, x_2 \geq 0 \text{ and integers.}$$

(b) Attempt any **one :** **4**

(i) Find (only) lower bound of the following IPP using Branch & Bound method.

Minimize $Z = 3x_1 + 2.5x_2$

Subject to the constraints :

$$x_1 + 2x_2 \geq 20$$

$$3x_1 + 2x_2 \geq 50; x_1, x_2 \geq 0 \text{ and integers.}$$

(ii) Discuss Branch & Bound method.

(c) Choose the correct **one** : **3**

(i) The use of cutting plane method

(a) reduce the number of constraints in the given problem.

(b) yields better value of objective function.

(c) require use of standard LP approach between each cutting plane application.

(d) none of these.

(ii) Branch & Bound method divides the feasible solution space into smaller parts by

(a) branching

(b) bounding

(c) enumerating

(d) all of the above.

(iii) Which of the following is the consequence of adding a new cut constraint to an optimal simplex table ?

(a) addition of new variable to the table.

(b) makes the previous solution infeasible.

(c) eliminates non-integer solution from the solution space.

(d) all of the above.

3. (a) Attempt any **one** : **7**

(i) What is the percentage change in optimal cost of transportation problem obtained by LCM or VAM ?

	P	Q	R	S	**Supply**
A	11	13	17	14	250
B	16	18	14	10	300
C	21	24	13	10	400
Demand	200	225	275	250	

(ii) Find the optimal assignment schedule.

	Location					
		P	Q	R	S	T
Machine	A	9	11	15	10	11
	B	12	9	–	10	9
	C	–	11	14	11	7
	D	14	8	12	7	8

(b) Attempt any **one :** **4**

(i) Explain the characteristics of opportunity costs.

(ii) Discuss the steps to be performed in solving assignment problem with objective of maximization ?

(c) Choose the correct **one :** **3**

(i) One disadvantage of NWCM to find initial solution to the transportation problem is that

(a) it is complicated to use

(b) it does not take into account cost of transportation

(c) it leads to a degenerate initial solution

(d) all of the above.

(ii) During an iteration while moving from one solution to the next, degeneracy may occur when

(a) the closed path indicates a diagonal move.

(b) two or more occupied cells are on the closed path but neither of them represents a corner of the path.

(c) two or more occupied cells on the closed path with minus sign are tied for lowest circled value.

(d) either of the above.

(iii) The purpose of a dummy row or column in an assignment problem is to

(a) obtain balance between total activities and total resources.

(b) prevent a solution from becoming degenerate.

(c) provide a means of representing a dummy problem.

(d) none of the above.

4. (a) Attempt any **one** : **7**

(i) Obtain necessary conditions (only) for the optimum solution of

Minimize $Z = 3e^{2x_1+1} + 2e^{x_2+5}$

Subject to the constraints :

$x_1 + x_2 = 7; x_1, x_2 \geq 0.$

(ii) Use Wolfe's method to obtain the modified QP (auxiliary equations) for following non-linear programming problem.

Maximize $Z = 2x_1 - x_1^2 + x_2$

Subject to the constraints :

$2x_1 + 3x_2 \leq 6$

$2x_1 + x_2 \leq 4; x_1, x_2 \geq 0.$

(b) Attempt any **one** : **4**

(i) Write the matrix associated with quadratic form $2x_1^2 + 3x_2^3 - 5x_1x_2$. Comment on the matrix obtained.

(ii) A company sells two types of items A and B. Item A sells for ₹ 25/ unit and sales revenue for item B decreases as the number of its units sold increases and Sales revenue is given by Sales revenue = $30x_2 - 0.30x_2^2$. The marketing department has only 1200 hours available for distributing these items in the next year. Further, the company estimates the sales function as non-linear and is given by Sales time = $x_1 + 0.2x_1^2 + 3x_2 + 0.35x_2^2$. The company can only procure 6000 units of item A and B for sales in the next year. Formulate the problem to determine number of units of product A and B to be procured so as to maximize the total revenue.

(c) Attempt any **one** : **3**

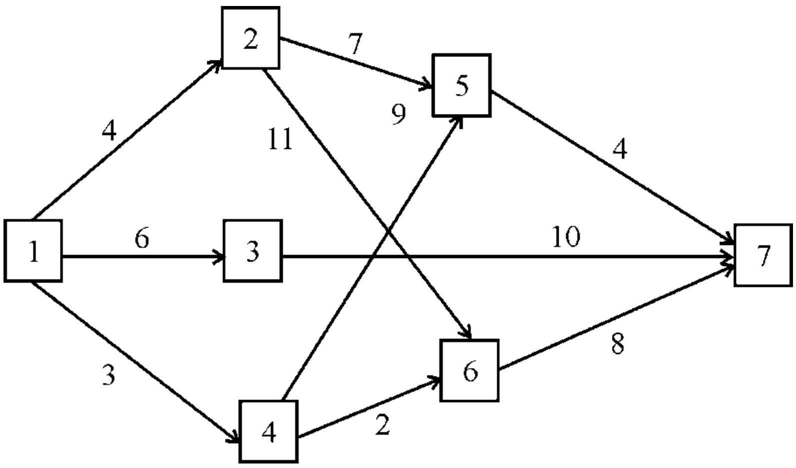
(i) State Kuhn-Tucker necessary and sufficient conditions in NLPP.

(ii) Comment upon the nature of the solution when

$$\left. \left| \frac{\partial z}{\partial x_{N_j}} \right| \right|_{x_N = 0} = \alpha_j \text{ in Beale's method.}$$

5. (a) Attempt any **one** : **7**

(i) Using dynamic programming, find the shortest path to be travelled by the tourist.



(ii) Maximize $\dfrac{x_1 + 2x_2 + 7}{2x_1 + x_2 + 13}$

Subject to :

$x_1 + x_2 \le 6$

$2x_1 + 5x_2 \le 10;\ x_1, x_2 \ge 0$

(b) Attempt any **one** : **4**

(i) Use dynamic programming approach to minimize

$$Z = \sum_{i=1}^{n} y_i^2 \text{ subject to } \prod_{i=1}^{n} y_i = c, c \neq 0, y_i \geq 0, j = 1, 2, \ldots n.$$

(ii) Define optimality criteria for fractional programming problem.

(c) Attempt any **one** : **3**

(i) Define

(1) State variable, (2) Stage

(ii) What is role of return function ?