Seat No. : _______________

NF-132

December-2015

M.Sc., Sem.-III

503 : Statistics

(Multivariate Analysis)

Time : 3 Hours] **[Max. Marks : 70**

Instructions : (1) **All** questions carry equal marks.

(2) Scientific calculator can be used.

1. (a) Let $\underline{x}_r$, r = 1, 2, …k, be independently distributed as $N_p(\underline{\mu}_r, \Sigma_r)$. Then for fixed matrices A_r : m x p, obtain the distribution of $\sum_{r=1}^{k} A_r \underline{x}_r$.

(i) If $\underline{\mu}_r = \underline{\mu}$ and $\Sigma_r = \Sigma$; r = 1, 2,…..k, then obtain the distribution of $\overline{\underline{x}}$.

(ii) If $\underline{x}_1$ and $\underline{x}_2$ are independently distributed as $N_p(\underline{\mu}, \Sigma)$; r = 1, 2, then obtain the distribution of $\underline{y}_1 = \underline{x}_1 + \underline{x}_2$ and $\underline{y}_2 = \underline{x}_1 - \underline{x}_2$. Also write the joint distribution of $\underline{y}_1$ and $\underline{y}_2$.

OR

(a) Let $\underline{x} \sim N_p(\underline{\mu}, \Sigma)$ and let $\underline{x}$, $\underline{\mu}$ and Σ be partition as follows :

$$\underline{x} = \begin{bmatrix} \underline{x}_1 \\ \underline{x}_2 \end{bmatrix}\begin{matrix} r \\ s \end{matrix}, \ \underline{\mu} = \begin{bmatrix} \underline{\mu}_1 \\ \underline{\mu}_2 \end{bmatrix}\begin{matrix} r \\ s \end{matrix} \text{ and } \Sigma = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{12} & \Sigma_{22} \end{bmatrix}\begin{matrix} r \\ s \end{matrix}, \ r + s = p.$$

(i) Show that $\underline{x}_1 - \Sigma_{22}^{-1} \Sigma_{12} \underline{x}_2$ and $\underline{x}_2$ are independently distributed.

(iii) Obtain the conditional distribution of $(x_1/x_2 = x_2)$

(b) Define canonical correlation coefficients and canonical variates. In usual notation show that the canonical correlation are solution of the determinant equation

$$\begin{vmatrix} -\lambda\Sigma_{11} & \Sigma_{12} \\ \Sigma_{12} & -\lambda\Sigma_{22} \end{vmatrix} = 0$$

Hence or otherwise, show that multiple correlation and simple correlation are special cases of canonical correlation.

OR

(b) Define partial correlation coefficient. In usual notation obtain the expression in terms of elements of $\Sigma^{-1} = (\sigma^{ij})$ for partial correlation coefficient. Show that partial correlation coefficient between x_1 and x_2 is nothing but the conditional correlation between x_1 and x_2 given $\underline{x}_3$.

2. (a) Define Wishart matrix. Obtain probability density function Wishart matrix V:pxp when $n \geq p$, $\underline{\mu} = 0$ and $\Sigma = I_p$.

OR

(a) Let $V \sim W_p(V, n, \Sigma)$, $n \geq p$, $\Sigma > 0$; and let $V = \begin{bmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{bmatrix}$, $\Sigma = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{bmatrix}$,

$V_{1.2} = V_{11} - V_{12} V_{22}^{-1} V_{12}'$, $\Sigma_{1.2} = \Sigma_{11} - \Sigma_{12} \Sigma_{22}^{-1} \Sigma_{12}'$ and $\beta = \Sigma_{12} \Sigma_{22}^{-1}$, with $r + s = p$. Then show that $V_{1.2}$ and (V_{12}, V_{22}) are independently distributed, where $V_{1.2} \sim W_r(\Sigma_{1.2}, n - s)$, V_{12} given V_{22} is $N_{r,s}(\beta V_{22}, \Sigma_{1.2}, V_{22})$, and $V_{22} \sim W_s(\Sigma_{22}, n)$.

(b) Define Hotelling's T^2 statistic. Show that it is used to test the $H_0 : \underline{\mu} = \underline{\mu}_0$ against $H_1 : \underline{\mu} \neq \underline{\mu}_0$. When $\underline{x} \sim N_p(\mu, \Sigma)$. Obtain the distribution of T^2 under H_0. What is the power of the test ?

OR

(b) Show that Hotelling's T^2 can be used to test $H_0 = \rho_{1.23.....p} = 0$ against $H_1 = \rho_{1.23.....p} \neq 0$, where $\rho_{1.23.....p}$ is multiple correlation coefficient.

3. (a) Discuss the problem of classifying an observation $\underline{x}_0$ into two known multivariate normal populations with common covariance matrix Σ. Write the classification rule in terms of the Mahalanobis distance Δ^2.

OR

(a) Obtain null distribution of the sample correlation coefficient 'r'.

(b) Define sample Mahalanobis distance D^2, obtain the relation between D^2 and Hotelling's T^2. Hence, obtain the distribution of D^2.

OR

(b) Obtain the estimated minimum ECM rule for classifying an object $\underline{x}_0$ when $\Sigma_1 = \Sigma_2$.

4. (a) Explain orthogonal factor model with K common factors. Give principal component solution of the factor model.

OR

(a) Explain the technique of One Way MANOVA for the comparison of several multivariate population means.

(b) Define GLM. Obtain MLE of $\underline{\beta}$ and σ^2 in GLM. How would you test H : $\underline{\beta} = \underline{\beta}_0$?

OR

(b) Define principal components. Write its important applications. If $\Sigma = \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}$ where $\rho > 0$, then find the principal components associated with matrix Σ and find the percentage of total variance explained by first principal component.

5. Answer the following questions :

(i) If $x = \begin{bmatrix} 6 & 10 & 8 \\ 9 & 6 & 3 \end{bmatrix}$ is an observation matrix of order 2×3, then obtain its mean vector and variance-covariance matrix.

(ii) If $\underline{x}$: 3×1 is distributed as $N_3(\underline{\mu}, \Sigma)$ with $\mu' = (2, 1, 1)$ and $\Sigma = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 3 & 2 \\ 1 & 2 & 2 \end{bmatrix}$, then obtain the distributions of $y_1 = x_1 + x_2$ and $y_1 = x_2 - x_3$. Write the joint distribution of y_1 and y_2.

(iii) Let x_1, x_2 and x_3 be distributed as $N_3(\underline{\mu}, \Sigma)$ with $\underline{\mu} = \underline{0}$ and $\Sigma = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 2 & 3 \\ -1 & 1 & 2 \end{bmatrix}$, then obtain

(i) $E(x_1 x_1' + x_2 x_2' + x_3 x_3')$

(ii) $E|(x_1 x_1' + x_2 x_2' + x_3 x_3')|$.

(iv) If the joint pdf of (x, y) is $\frac{1}{2\pi} \exp\left[\frac{-1}{2}\{(x-1)^2 + (y-2)^2\}\right]$, then write the values of μ_x, μ_y, σ_x, σ_y and ρ_{xy}.

(v) Let $\underline{x} \sim N_3(\underline{\mu}, \Sigma)$, where $\underline{\mu} = \underline{0}$ and $\Sigma^{-1} = \begin{bmatrix} -1 & 2 & -1 \\ 5 & -8 & 2 \\ -3 & 5 & -1 \end{bmatrix}$. Write the formula for multiple correlation coefficients $R_{1.23}$ and obtain its value.

(vi) Let $\underline{x} \sim N_p(\underline{\mu}, \Sigma)$. Obtain characteristic function of the vector $\underline{x}$.

(vii) Let $\underline{X}_1, \underline{X}_2, \ldots, \underline{X}_{20}$, be a random sample of size n = 20 from a $N_6(\underline{\mu}, \Sigma)$ population. Specify the distribution of B (19S) B, where $B = \begin{pmatrix} 1 & -1/2 & -1/2 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1/2 & -1/2 & 1 \end{pmatrix}$, where 'S' is the MLE's of Σ.