Seat No. : $\qquad$

# NG-111 <br> December-2015 <br> $4^{\text {th }}$ Year M.Sc., (CA \& IT) <br> <br> Operation Research 

 <br> <br> Operation Research}

Time : 3 Hours]
[Max. Marks : 100

Instruction : Graph paper statistical table will be provided on request.

## 1. Attempt any two :

(a) A company engaged in tinned food delicacies has 300 trained employees on its rolls each of whom can produce one tin of food in a week. Due to the developing taste of the public for this kind of food, the company plans to add the existing labour force by employing 150 people in a phased manner over the next five weeks. The newcomers would have to undergo a two week training program before being put to work. The training is to be given by employees from amongst the existing ones and it is known that one employee can train three employees. Assume that there would be no production coming forth from the trainers and the trainees during training period as the training is off-th-job. However, trainees would be remunerated at the rate of ₹ 300 per week, the same rate as is for the trainers.

The company has booked the following number of tins to supply during the next five weeks :

| Week : | 1 | 2 | 3 | 4 | 5 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| No. of tins : | 280 | 298 | 305 | 360 | 400 |

Assume that the production in any week would not be more than the number of tins ordered for so that every delivery of the delicacy would be 'fresh'.

Formulate it as a linear programming problem to develop a training schedule that will minimize the labour costs over the five week period.
(b) Solve the following problem graphically:

$$
\operatorname{Min} Z=90 x_{1}+60 x_{2}
$$

subject to

$$
\begin{aligned}
& 20 x_{1}+30 x_{2} \geq 900 \\
& 40 x_{1}+30 x_{2} \geq 1200 \\
& x_{1}, x_{2} \geq 0
\end{aligned}
$$

(c) The initial simplex table of an LPP is given below :

| $\mathrm{C}_{\mathrm{j}}:$ |  | 20 | 30 | 0 | 0 | -M | -M |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Basic <br> Variable |  | $\mathrm{X}_{1}$ | $\mathrm{X}_{2}$ | $\mathrm{~S}_{1}$ | $\mathrm{~S}_{2}$ | $\mathrm{~A}_{1}$ | $\mathrm{~A}_{2}$ | Solution |
| $\mathrm{A}_{1}$ | -M | 8 | 6 | -1 | 0 | 1 | 0 | 24 |
| $\mathrm{~S}_{2}$ | O | 8 | 5 | 0 | 1 | 0 | 0 | 30 |
| $\mathrm{~A}_{2}$ | -M | 3 | -2 | 0 | 0 | 0 | 1 | 18 |

(i) Write down the original problem represented by this table.
(ii) Is the solution given in the table optimal ? If optimal then given reason why it is optimal. If not optimal find out the optimal solution.
2. (a) Define the following :
(i) Unbalanced transportation Problem
(ii) Prohibited Route
(iii) Degeneracy in transportation Problem
(iv) Transshipment problem
(b) A product is produced by 4 factories $\mathrm{F}_{1}, \mathrm{~F}_{2}, \mathrm{~F}_{3}$ and $\mathrm{F}_{4}$. Their unit production costs are ₹ $2,3,1$ and 5 respectively. Production capacity of the factories are 50, 70, 30 and 50 units respectively. The product is supplied to 4 stores $S_{1}, S_{2}, S_{3}$ and $S_{4}$, the requirements of which are $25,35,105$ and 20 respectively.
Unit costs of transportation are given below :

|  | $\mathrm{S}_{1}$ | $\mathrm{~S}_{2}$ | $\mathrm{~S}_{3}$ | $\mathrm{~S}_{4}$ |
| :---: | :---: | :---: | :---: | :---: |
| $\mathrm{~F}_{1}$ | 2 | 4 | 6 | 11 |
| $\mathrm{~F}_{2}$ | 10 | 8 | 7 | 5 |
| $\mathrm{~F}_{3}$ | 13 | 3 | 9 | 12 |
| $\mathrm{~F}_{4}$ | 4 | 6 | 8 | 3 |

Find the transportation plan such that the total production and transportation cost is minimum. [Use least cost method for initial solution]

## OR

(b) The following is the transshipment problem with 4 sources and 2 destinations. The supply values of the sources $S_{1}, S_{2}, S_{3}$ and $S_{4}$ are $100,200,150$ and 350 units respectively. The demand values of destinations $D_{1}$ and $D_{2}$ are 350 and 450 units respectively Transportation cost per unit between various defined sources and destinations are given in the following table.
Solve the transshipment problem :

|  | $\mathrm{S}_{1}$ | $\mathrm{~S}_{2}$ | $\mathrm{~S}_{3}$ | $\mathrm{~S}_{4}$ | $\mathrm{D}_{1}$ | $\mathrm{D}_{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{~S}_{1}$ | 0 | 4 | 20 | 5 | 25 | 12 |
| $\mathrm{~S}_{2}$ | 10 | 0 | 6 | 10 | 5 | 20 |
| $\mathrm{~S}_{3}$ | 15 | 20 | 0 | 8 | 45 | 7 |
| $\mathrm{~S}_{4}$ | 20 | 25 | 10 | 0 | 30 | 6 |
| $\mathrm{D}_{1}$ | 20 | 18 | 60 | 15 | 0 | 10 |
| $\mathrm{D}_{2}$ | 10 | 25 | 30 | 23 | 4 | 0 |

3. (a) Formulate the following assignment problem as a linear programming model for assigning program to programmer to complete the programs in minimum time. Table gives the time required for completing program by programmer. $\quad \mathbf{1 0} \times \mathbf{2}=\mathbf{2 0}$

## Programmer

| Program | A | B | C | D | E |
| :---: | :---: | :---: | :---: | :---: | :---: |
| I | 120 | 100 | 80 | 90 | 130 |
| II | 80 | 90 | 110 | 70 | 110 |
| III | 110 | 140 | 120 | 130 | 160 |

(b) The marketing manager has 5 salesmen and there are 5 sales districts. Considering the capabilities of the salesmen and the nature of districts, the estimates made by the marketing manager for the sales per month (in ₹ 1,000 ) for each salesman in each district would be as follows :

|  | A | B | C | D | E |
| :--- | :---: | :---: | :---: | :---: | :---: |
|  | 32 | 38 | 40 | 28 | 40 |
| 2 | 40 | 24 | 28 | 21 | 36 |
| 3 | 41 | 27 | 33 | 30 | 37 |
| 4 | 22 | 38 | 41 | 37 | 36 |
| 5 | 29 | 33 | 40 | 35 | 39 |

Find the assignment of salesmen to districts that will result in the maximum sales.
OR
(b) A trip from Chennai to Bengaluru takes six hours by bus. A typical time table of the bus service in both directions is given below :
Departure Route Arriving at
from Chennai number Bengaluru

| $06-00$ | a | $12-00$ |
| :--- | :--- | :--- |
| $07-30$ | b | $13-30$ |
| $11-30$ | c | $17-30$ |
| $19-00$ | d | $01-00$ |
| $00-30$ | e | $06-30$ |

NG-111

| Departure | Route | Arriving at |
| :---: | :---: | :---: |
| from | number | Chennai |

## Bengaluru

| $05-30$ | 1 | $11-30$ |
| :--- | :--- | :--- |
| $09-00$ | 2 | $15-00$ |
| $15-00$ | 3 | $21-00$ |
| $18-30$ | 4 | $00-30$ |
| $00-00$ | 5 | $06-00$ |

The cost of providing this service by the transport company depends upon the time spent by the bus crew (driver and conductor) away from their places in addition to service time. There are five crew. There is a constraint that every crew should be provided with more than 4 hours of rest before the return trip again and should not wait for more than 24 hours for the return trip. The company has residential facilities at both the places. Find which crew should be assigned with which line of service or which service line should be connected with which other line so as to reduce the waiting time to the minimum.
4. (a) A project is represented by the network shown below and has the following table : 20 20
$\begin{array}{llllllllllllll}\text { Activity } & \text { A } & \text { B } & \text { C } & \text { D } & \text { E } & \text { F } & \text { G } & \text { H } & \text { I } & \text { J } & \text { K } & \text { L } & \text { M }\end{array}$

| Predecessor | - | - | B | A | C | C | F | F | H | I | D, E, | I | K, L |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Duration | 6 | 4 | 10 | 1 | 1 | 3 | 14 | 6 | 9 | 2 | 7 | 5 | 8 |

(i) Draw the network
(ii) What is project completion time ?
(iii) Using free float and total float, find the critical activity

## OR

(a) Consider the project having following activities with their estimated time.
(i) Draw the network diagram for the project.
(ii) Compute the expected project completion time.
(iii) What should be the due date to have 0.90 probability of completion?

|  |  | Activity Time (Weeks) |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Activity | Immediate | Most <br> Predecessor | Most <br> Optimistic <br> Likely | Most <br> Pessimistic |
| A | - | 3 | 4 | 5 |
| B | - | 4 | 8 | 10 |
| C | B | 5 | 6 | 8 |
| D | A, C | 9 | 15 | 10 |
| E | B | 4 | 6 | 8 |
| F | D, E | 3 | 4 | 5 |
| G | D, E | 5 | 6 | 8 |
| H | D, E | 1 | 3 | 4 |
| I | G | 2 | 4 | 5 |
| J | F, I | 7 | 8 | 10 |
| K | G | 4 | 5 | 6 |
| L | H | 8 | 9 | 10 |
| M | J, K, L | 6 | 7 | 8 |

5. Answer any two :
$10 \times 2=20$
(a) Consider the following data of a flow network as shown below and formulate it as LPP to find the maximum flow from node 1 to node 6 .

|  | Flow |  | Flow |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Arc $\mathrm{i}-\mathrm{j}$ | $\mathrm{f}_{\mathrm{ij}}$ | $\mathrm{f}_{\mathrm{ji}}$ | $\operatorname{Arc} \mathrm{i}-\mathrm{j}$ | $\mathrm{f}_{\mathrm{ij}}$ | $\mathrm{f}_{\mathrm{ji}}$ |
| $1-2$ | 60 | 10 | $3-4$ | 35 | - |
| $1-3$ | 35 | 25 | $3-5$ | 30 | 28 |
| $2-3$ | 25 | 20 | $4-5$ | 45 | - |
| $2-4$ | 19 | 24 | $4-6$ | 40 | - |
| $2-5$ | 25 | 30 | $5-6$ | 55 | - |
|  |  |  |  |  |  |

(b) In a large soap product plant, quality control inspectors take samples of various products from the different production areas and deliver them to the lab for analysis. The inspection process is slow and the inspectors spend substantial time transporting samples from the production areas to the lab. The company is considering installing a conveyor belt system that could transport the samples between the production areas and the lab. The following network shows the location of the lab and production areas (nodes) where sample must be collected. Arcs show the connectivity among the nodes along with the distances in 100 ft . Find out the layout of the conveyor system that will enable the production centres to spend samples to the lab.

(c) A company produces variety of candy products. Company trucks are used to deliver local orders directly to retail outlets. When business was small the drivers of the trucks were free to take routes of their choice. As the business has grown transportation and delivery cost have become significant. In an effort to improve this the manager wants you to find out which route a truck should take from node 1 to node 11 . Following is the network of routes with the time of transportation :


