Seat No. : _______________

NH-101

November-2013

B.Sc. Sem.-III (CBCS)

STA-201 : Statistics

Time : 3 Hours] **[Max. Marks : 70**

Instruction : **All** questions carry equal marks.

1. (a) Define random variable, distribution function and state and prove properties of distribution function.

OR

Let X be a continuous random variable with p.d.f. $f(x)$ and $y = g(x)$ is strictly monotonic function. If $g(x)$ is differentiable for all x then prove that p.d.f. $h(\cdot)$ of y is given by $h_y(y) = f(x)\left|\frac{dy}{dx}\right|$.

(b) Show that geometric mean G of the distribution with p.d.f.

$f(x) = 6(2 - x)(x - 1)\ ;\ 1 \leq x \leq 2$

is given by $6\, ln\,(16G) = 19$.

OR

A function $f(x)$ is given as under

$$f(x) = \begin{cases} x & \text{if } 0 \leq x \leq 1 \\ \frac{3-x}{4} & \text{if } 1 < x \leq 3 \\ 0 & \text{o.w.} \end{cases}$$

Can $f(x)$ be p.d.f ? If so obtain its distribution function. Hence find $P_r\left(\frac{1}{2} < x < \frac{3}{2}\right)$.

Also find mean.

2. (a) Define expectation of a r.v. and state and prove addition theorem of expectation of two continuous random variables. If r.v. X and Y are independent then prove that $E(XY) = E(X) \cdot E(Y)$.

OR

Define cumulant generating function and explain effect of change of origin and scale on cumulants. Also prove that the r^{th} cumulant of the sum of the independent random variables is equal to the sum of r^{th} cumulants of the individual variables.

(b) If X is a random variable having cumulants k_r; (r = 1, 2, …) given by $k_r = (r - 1)!\, pa^{-r}$ (p, a > 0) then find m.g.f. of X. If r^{th} moment about origin of a r.v. X is r ! then find m.g.f. of X.

OR

Find the expected value of $3x^2 - 1$ if the p.d.f. of X is

$$f(x) = 2x \quad ; \quad 0 < x < 1$$
$$= 0 \quad ; \text{ o.w.}$$

If X and Y are independent variates with mean 10 and 20 and variances 2 and 3 respectively then find Var(3*x* + 4y).

3. (a) If X ~ B(n, p) then obtain the expression for r^{th} factorial moment about origin. Hence find first three moments about origin and central moments.

OR

Stating the conditions obtain probability function of Poisson distribution from Binomial distribution.

(b) If X ~ P(1) then show that mean deviation about mean is $\frac{2}{C}$ times the standard deviation.

OR

Obtain recurrence relation for the central moments of Binomial distribution.

4. (a) Define Beta distribution of second kind. Also find r^{th} moment about origin, mean, variance and harmonic mean for Beta distribution of second kind.

OR

If X_1, X_2, … X_k are independent random variables having an exponential distribution with parameter Q_i; i = 1, 2, …, k then obtain the distribution of $Y = \min\{X_1, X_2, \ldots, X_k\}$.

(b) If X has a rectangular distribution in (0, 1) then find the probability function of $Y = -2 \log_e X$. Also find E(Y).

OR

Find m.g.f. about origin for a rectangular distribution with p.d.f.

$$f(x) = \frac{1}{2a} \quad ; \quad -a < x < a$$
$$= 0 \quad ; \quad \text{o.w.}$$

Also show that even ordered central moments are given by $\mu_{2n} = \frac{a^{2n}}{2n + 1}$.

5. (1) What is the relation between $M_x(t)$ and $M_{\frac{x-\mu}{\sigma}}(t)$?

(2) If function f(x) defined as

$$f(x) = (x) \;;\; -1 \leq x \leq 1$$
$$= 0 \;;\; \text{o.w.}$$

a p.d.f. of a random variable x. Give reason.

(3) Define two dimensional random variable.

(4) If the p.d.f. of a random variable x is

$$f(x) = ke^{-2x} \;;\; x \geq 0$$
$$= 0 \;;\; \text{o.w.}$$

then find the value of k.

(5) What is the relation between mean and harmonic mean of Beta distribution of second kind ?

(6) Define independence of two random variables.

(7) If X ~ B (8, 0.3) then what is the m.g.f. of X ?

(8) What is the recurrence relation for central moments of Poisson distribution ?

(9) Write m.g.f. of exponential distribution and find its mean.

(10) If X has rectangular distribution in (a, b) with mean = 1 and variance = $\frac{4}{3}$ then find a and b.