Seat No. : _______________

DD-107

December-2013

B.C.A. Semester – I

CC-104 : Basics of Mathematics (BM)

Time : 3 Hours] **[Max. Marks : 70**

1. (A) (a) Let $U = \{x \in Z / 0 < x^2 < 32\}$, $A = \{2, 3, 4\}$ and $B = \{-5, -3, -1, 2, 4\}$ then find, (i) $(A \cup B)'$ (ii) $(A - B) \cup B'$ **6**

(b) Let $A = \{x \in N - \{1, 2\} / x$ is an odd number less than $10\}$ and $B = \{1, 2, 3, 4, 7, 8, 10\}$ then find $A \Delta B$.

(c) Let $f(x) = \dfrac{x-1}{x+1}$, then find $f\left(\dfrac{1}{2}\right)$ and $f\left(\dfrac{1}{x}\right)$.

OR

(a) If $A = \{x \in Z / 1 < x < 7\}$, $B = \{x \in N / (x + 1)^2 < 50\}$ and $C = \{x \in Z / 0 < x < 10\}$. Verify that $(A \cup C) \Delta (B \cup A) = (B \Delta C) \cup A$.

(b) If $A \subset B$, then show that $B' \subset A'$.

(c) Let $f(x) = x^2 - 2x$ then find $f(x) + f(x + 1)$ for $x = 2$.

(B) (a) Give an example of sets A, B and C such that **4**

(i) $A \cap B = A \cap C$; but $B \neq C$.

(ii) $A \cup B = A \cup C$; but $B \neq C$.

(b) If $A = \{1, 2, 3\}$ and $B = \{a, b, c\}$, then find $A \times B$ and $B \times A$.

OR

(a) If $n(A) = 24$, $n(B) = 36$ and $n(A \cup B) = 50$, find $n(A \cap B)$.

(b) If $n(A) = 17$, $n(A \cup B) = 38$ and $n(A \cap B) = 2$, find $n(A - B)$, $n(B)$ and $n(B - A)$.

(C) (a) Let $f: R - \{-1\} \to R$, $f(x) = \left(\dfrac{1-x}{1+x}\right)$ then find the value of $f(x) + f(1/x)$ and $f(f(0))$. **4**

(b) Let $f(x) = \log_{10}x$ then find $\dfrac{f(100) + f(1000)}{f(10)}$.

OR

(a) If $f: R \to R$ and $g: R \to R$, $f(x) = x + 1$ and $g(x) = 2x - k$ and $fog = gof$ then find k.

(b) Give Domain and Range for the function $f: Z \to N$, $f(x) = |x| + 1$.

2. (A) For given matrices $A = \begin{bmatrix} 2 & 4 & 3 \\ -3 & 2 & 0 \\ -1 & 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ **6**

(a) Find 3A – B.

(b) Find (AB).

(c) Find the rank of a matrix (AB).

OR

For the given matrix $A = \begin{bmatrix} 2 & -1 & 3 \\ 4 & 2 & 1 \\ 3 & 1 & 1 \end{bmatrix}$

(a) Find the determinant of a matrix A.

(b) Find the rank of a matrix A.

(c) Find the inverse of A by the definition of inverse of a Matrix.

(B) Express the given matrix $A_{3 \times 3}$ as a sum of a symmetric and a skew-symmetric matrices. **4**

$$A = \begin{bmatrix} 2 & 1 & -1 \\ 2 & 2 & 2 \\ -1 & -2 & -2 \end{bmatrix}$$

OR

For a given matrix $A = \begin{bmatrix} 1 & 2 & 0 \\ 3 & -1 & 4 \end{bmatrix}$ find AA^T and A^TA.

(C) Solve the following system using Cramer's Rule. **4**

$$x + 2y + 2z = 5$$
$$3x + 2y + z = 6$$
$$x + 2y + 3z = 7$$

OR

Solve the following system using inversion method.

$$x + y + z = 3$$
$$x + 2y + 3z = 6$$
$$3x + y + 2z = 6$$

3. (A) (a) Find the distance between two points (–1, –2) and (4, 5). **6**

(b) If the point $(x, 2)$ is equidistance from (8, –2) and (2, 2), find the value of x.

(c) Show that three points (1, 1), (2, 2) and (3, 3) are collinear.

OR

(a) What will be the value of x if the distance between $(x, 4)$ and (–5, 4) be 10 ?

(b) Find the area of a triangle formed by three points (1, 1), (2, 4) and (5, 2).

(c) If the distance between A(5, a) and B(2, 6) is $3\sqrt{2}$, find the value of a.

(B) (a) If a point P(1, 2) divides a line segment joining points A (–2, –1) and B in the ratio 2 : 3 then find the x-coordinate of point B. **4**

(b) Give an equation of a line having y – intercept 3 and slope 2.

OR

(a) Determine x so that 5 is the slope of the line through (x, 12) and (3, 2).

(b) Find the equation of a line which cuts off equal intercepts and passes through (3, 5).

(C) A(3, 4) and B(5, –2) are the two points. Find the point P such that PA = PB and area of ΔPAB = 10. **4**

OR

Find the equations of two lines passing through the point (2, –1) and making an angle of 45° with the line $6x + 5y - 1 = 0$.

4. (A) (a) Find $\lim\limits_{x \to 2} \dfrac{x^7 - 128}{x - 2}$ **6**

(b) Find $\dfrac{dy}{dx}$ for $y = x^3 - \log x$

(c) Evaluate : $\int (x^2 + 2x + 1)\, dx$

OR

(a) Check the continuity of f(x) at $x = 5$.

$$f(x) = \begin{cases} \dfrac{x^2 - 9}{x - 3}, & x < 3 \\ = 6, & x \geq 3 \end{cases}$$

(b) Find derivative of $y = x^3 + e^x$ w.r.t. x.

(c) Evaluate : $\int \dfrac{1}{2x + 7}\, dx$

(B) (a) Find $\dfrac{dy}{dx}$ when $y = x^4 2^x e^x$ **4**

(b) Evaluate : $\int \left(t^2 + 2t + \dfrac{1}{t^2}\right) dt$

OR

(a) Find $\dfrac{dy}{dx}$ when $y = x \cdot e^x$

(b) Evaluate : $\int (2\sec x \tan x)\, dx$

(C) (a) Find $\frac{dy}{dx}$ when $y = e^{3x + 4}$ **4**

(b) Evaluate : $\int_1^2 \frac{\log_2 x}{x} \, dx$

OR

(a) Find $\frac{dy}{dx}$ when $y = \sin^5 x$

(b) Evaluate $\int_0^1 (x^2 + 5) \, dx$

5. Do as directed. **14**

(1) Write the Set A = {2, 4, 6, 8, …, 20} by Property method.

(2) Give the Range for the function $f: N \to N$, $f(x) = x$.

(3) List the elements of the set $A = \{x / x^4 - x = 0, x \in N\}$.

(4) Power set of $A = \{\mu, \lambda, \sigma\}$ has 9 elements. (True / False)

(5) For any matrix A the matrix A + AT is a symmetric matrix. (True / False)

(6) For any matrix A, $AA^{-1} = I$. (True / False)

(7) Give an example of a matrix A such that $A^T = -A$.

(8) Find the slope of a line $x + y + 1 = 0$.

(9) Give an equation of a line passing through points (2, 0) and (3, 0).

(10) Two lines $x - y = 0$ and $x + y = 0$ are perpendicular. (True / False).

(11) Find : $\lim_{x \to 2} \frac{x^2 + 2x}{x}$.

(12) Is the function $f: R \to R$, $f(x) = x$ continuous at $x = 2$?

(13) For $y = e^x$ find $\frac{d^2y}{dx^2}$.

(14) Evaluate the integration of the function $y = 2^2 + 3^3 + \pi$ with respect to x.