Seat No. : _______________

# DB-105

**December-2013**

**B.Sc. (CBCS) Sem.-V**

**MAT-304 : Mathematics**

**(Mathematical Programming)**

**Time : 3 Hours]** **[Max. Marks : 70**

**Instructions :** (1) **All** questions are compulsory.
(2) Each question carries equal marks.

1. (a) Define a convex set. Prove that the intersection of two convex sets is also convex. Is the union of two convex sets, a convex ? Justify your answer. **4**

(b) Attempt any **two** : **10**

(1) Consider a set $S = \{\bar{x} \in E^n / \|\bar{x}\| = 1\}$. Is a set S convex ? Justify.

(2) Show that the set of all convex combinations of a finite numbers of points of $S \subset E^n$ is a convex.

(3) A manufacturer produces two different models X and Y of the same product. Model X makes a contribution of ₹ 50 per unit and Model Y ₹ 30 per unit towards total profit. Raw materials $r_1$ and $r_2$ are required for production. At least 18 kg of $r_1$ and 12 kg of $r_2$ must be used daily. Also at most 34 hours of labour are to be utilized. A quantity of 2 kg of $r_1$ is needed for model X and 1 kg of $r_1$ for model Y. For each X and Y, 1 kg of $r_2$ is required. It takes 3 hours to manufacture model X and 2 hours to manufacture model Y. A manufacturer wishes to maximize the profit. Formulate the linear programming problem.

2. (a) Define a feasible solution. If $S_F$ is a non-empty set of all feasible solutions of a L.P. problem then prove that $S_F$ is a convex set. **4**

(b) Attempt any **two** : **10**

(1) Solve the following L.P. problem by Simplex method :

Maximize $Z = 3x_1 + 2x_2 + 5x_3$

Subject to
$$2x_1 + 3x_2 \le 8$$
$$2x_1 + 5x_2 \le 10$$
$$3x_1 + 2x_2 + 4x_3 \le 15$$
and $x_1, x_2 \ge 0$

(2) Solve the following L.P. problem by Big-M method or two-phase simplex method :

Minimize $Z = x_1 + x_2$

Subject to $2x_1 + x_2 \geq 4$

$x_1 + 7x_2 \geq 7$

and $x_1, x_2 \geq 0$

(3) Solve the following integer programming problem by the Cutting plane method :

Maximize $Z = x_1 + x_2$

Subject to $3x_1 + 2x_2 \leq 5$

$x_1 \leq 2$ ;

$x_1, x_2 \geq 0$ and are integers.

3. (a) Define a Dual of a primal. Prove that the dual of the dual is a primal. **4**

(b) Attempt any **two** : **10**

(1) Using the principle of duality, solve the following L.P. problem :

Maximize $Z = 4x_1 + x_2$

Subject to $x_1 + x_2 \geq 3$

$x_1 - x_2 \geq 2$

and $x_1, x_2 \geq 0$.

(2) Describe the solution of the following L.P. problem by solving its dual :

Maximize $Z = 3x_1 + 2x_2$

Subject to $2x_1 + x_2 \leq 5$

$x_1 + x_2 \leq 3$

and $x_1, x_2 \geq 0$

(3) Use the dual simplex method to solve the following L.P. problem :

Minimize $Z = 10x_1 + 6x_2 + 2x_3$

Subject to $-x_1 + x_2 + x_3 \geq 1$

$3x_1 + x_2 - x_3 \geq 2$

and $x_1, x_2, x_3 \geq 0$.

4. (a) Prove that the number of basic variables in transportation problem are at the most 'm – n + 1'. **4**

(b) Attempt any **two** : **10**

(1) Solve the following T.P. for minimum transportation cost of MODI's method :

| | $W_1$ | $W_2$ | $W_3$ | $W_4$ | **Supply** |
|---|---|---|---|---|---|
| $O_1$ | 21 | 16 | 25 | 13 | **11** |
| $O_2$ | 17 | 18 | 14 | 23 | **13** |
| $O_3$ | 32 | 27 | 18 | 41 | **19** |
| **Demand** | **6** | **10** | **12** | **15** | **43** |

(2) Find the optimum solution of the following T.P. :

| | $D_1$ | $D_2$ | $D_3$ | **Supply** |
|---|---|---|---|---|
| $O_1$ | 5 | 9 | 9 | **76** |
| $O_2$ | 17 | 25 | 17 | **82** |
| $O_3$ | 9 | 17 | 25 | **77** |
| **Demand** | **72** | **102** | **41** | |

(3) Solve the following $4 \times 4$ assignment problem to minimize the total cost :

| | A | B | C | D |
|---|---|---|---|---|
| I | 40 | 35 | 38 | 41 |
| II | 42 | 35 | 34 | 40 |
| III | 38 | 34 | 34 | 37 |
| IV | 39 | 36 | 38 | 36 |

5. (a) Answer in short : (each of **two** marks) **8**

(1) Show that [a, b] in R is convex set.

(2) (i) Illustrate a L.P. problem of two variables having no solution.

(ii) Illustrate a L.P. problem of two variables having infinitely many solution.

(3) Write down the dual of the following L.P. Problem :

Minimize $Z = 2x_1 + x_2 - 3x_3$

Subject to $2x_1 + x_2 - 3x_3 \le -5$

$x_1 + 2x_2 = 7$

$x_1 + x_3 \ge 9$

$-2x_1 - 5x_2 \ge -8$

and $x_1, x_2 \ge 0$ ;

$x_3$ – unrestricted variable.

(4) Determine an initial basic feasible solution of the following transportation problem using North-West corner method :

| | P | Q | R | S | **Supply** |
|---|---|---|---|---|---|
| A | 19 | 30 | 50 | 10 | **7** |
| B | 70 | 30 | 40 | 60 | **9** |
| C | 40 | 8 | 70 | 20 | **18** |
| **Demand** | **5** | **8** | **7** | **14** | **34** |

(b) Fill in the blanks with appropriate answer : (each of one mark) **6**

(1) For the maximization L.P. problem, the simplex method is terminated when all values of __________.

(a) $c_j - z_j \geq 0$ (b) $c_j - z_j \leq 0$

(c) $c_j - z_j = 0$ (d) $z_j \leq 0$

(2) If dual has an unbounded solution, primal has __________.

(a) no feasible solution (b) feasible solution

(c) unbounded solution (d) none of these

(3) Each constraint in L.P. problem is expressed as an __________

(a) inequality with ≥ sign (b) inequality with ≤ sign

(c) equality with = sign (d) none of these

(4) A constraint in L.P. problem restricts __________.

(a) value of objective function

(b) value of a decision variable

(c) use of the available resource

(d) all of these

(5) The dummy source or dummy destination in a transportation problem is added to __________.

(a) ensure that total cost does not exceed a limit

(b) prevent solution from becoming degenerate

(c) satisfy rim conditions

(d) none of these

(6) An assignment problem is considered as a particular case of a transportation problem because __________.

(a) all $x_{ij}$ = 0 or 1 (b) the no. of rows equal column

(c) all rim conditions are 1 (d) all of these