Seat No. : _______________

DA-134

December-2013

5 Years M.Sc. Sem-III

Discrete Mathematics S Y M.Sc.

(CA-IT) Integrated (KS)

Time : 3 Hours] **[Max. Marks : 100**

Instruction : Calculators are not allowed.

1. Attempt any **four** : **20**

(1) Define PCNF and PDNF. Obtain PCNF and PDNF of the formula

$(7P \to R) \land (Q \rightleftarrows P)$.

(2) Show that the conclusion 7P follows from the premises $R \to 7Q$, $R \lor S$, $S \to 7Q$, $P \to Q$ using indirect method of proof if needed.

(3) Show that $\{7, \to\}$ is functionally complete.

(4) Show that the following premises are inconsistent : $P \to Q$, $Q \to R$, $S \to 7R$ and $P \land S$.

(5) Define :

(i) Statement

(ii) Atomic Statement

(iii) Molecular Statement

If the truth values of P and Q are T and that of R and S are F, then find out the truth value of the following formula :

$(7 (P \land Q) \lor 7R) \lor (((7P \land Q) \lor 7R) \land S)$

2. Attempt any **four** : **20**

(1) Let $X = \{1, 2, 3, 4, 5\}$ and $R = \{\langle x, y\rangle \mid x - y$ is divisible by $2\}$. Show that R is an equivalence relation. Draw the graph of R and find M_R.

(2) Define any 4 types of retations giving examples.

Let $M_R = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$

What are the properties of relation R ?

(3) Show that $(Fx)\ M(x)$ follows logically from the premises $(x)\ (H(x) \rightarrow M(x))$ and $(Fx)\ H(x)$

(4) State one advantage of predicate calculus over statement calculus. Write the following statements using predicate formulas :

(1) Every two wheeler is a scooter.

(2) Every two wheeler that is a scooter is manufactured by Bajaj.

(3) There is a two wheeler that is not manufactured by Bajaj.

(4) There is a two-wheeler manufactured by Bajaj that is not a scooter.

(5) Let X = {1, 2, 3, 4} and a relation R on X defined by R = {<1, 2>, <4, 3>, <2, 2>, <2, 1>, <3, 1>}

Is R transitive ? Reflexive ? Symmetric ? Find a set $S \supseteq R$ such that S is transitive. Justify.

3. Attempt any **five** : **20**

(1) Define transitive closure of a relation.

Let X = {1, 2, 3}

R = {<1, 2>, <2, 3>, <3, 1>}. Find the transitive closure of R.

(2) Draw the Hasse diagram for the relation <P(A), ≤>, where A = {a, b, c, d} and P(A) denotes the power set of A.

(3) What is a partition ?

Let A = {a, b, c, d, e, f, g, h}. Consider the following subsets of A :

A_1 = {a, b, c, d}, A_2 = {a, c, e, f, g, h}

A_3 = {a, c, e, g}, A_4 = {b, d} and A_5 = {f, h}

Which of the following are partitions of A ? {A_1, A_2}, {A_1, A_5} and {A_3, A_4, A_5}. Justify.

(4) Draw the Hasse diagram of (S_{36}, D) where S_{36} denotes the set of positive divisors of 36 and D is relation "divides". Find the smallest and largest element of (S_{36}, D) if it exists.

(5) Let {L, ≤} be a lattice. Show that for any a, b, c ∈ L

$$b \le c \Rightarrow \begin{cases} a * b & \le \quad a * c \\ a \oplus b & \le \quad a \oplus c \end{cases}$$

Define a complemented lattice.

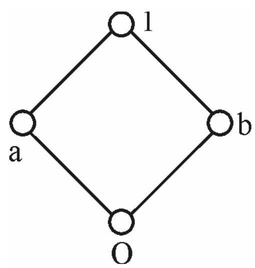
4. Attempt any **four** : **20**

(1) Let X = {1, 2, 3}, Y = {a, b}. List all the functions from X to Y and indicate in each case whether the function is one-one, onto and is one-one onto. If X has m elements and Y has n elements, how many functions are possible from X to Y ?

(2) What is an atom and antiatom ? Obtain the product of sums canonical form of the Boolean expression $x_1 * x_2$ having three variables x_1, x_2 and x_3.

(3) Represent and minimize the given Boolean expression using a K-map and then sketch its circuit diagram using necessary logic gates.

(4) Give an expression $\alpha(x_1, x_2, x_3)$ defined to be ⊕ 0, 3, 5, 7 and determine the value of α(a, b, 1) where a, b, 1 ∈ B and <B, ∗, ⊕, 0, 1> is the Boolean algebra given by



(5) Show that in a Boolean algebra for any a, b.

(1) a = b ⇔ (a ∗ b') ⊕ (a' ∗ b) = 0

(2) a = 0 ⇔ (a ∗ b') ⊕ (a' ∗ b) = b

(3) a ⊕ (a' ∗ b) = a ⊕ b

(4) a ∗ (a' ⊕ b) = a ∗ b

Define a Boolean Algebra.

5. Attempt any **four** : **20**

(1) Define a group giving examples. Draw binary operation tables for groups of order 1, 2 and 3. Are they abelian ? Give an example of a group which is not abelian.

(2) Let S = {1, 2, 3}. Find all permutations on the set S. Prepare a composition table for the same.

(3) Define a permutation. What is a permutation group ?

If $P = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & 6 & 2 & 4 & 1 & 3 \end{pmatrix}$,

express it in the form of product of disjoint cycles. Is it an even or an odd permutation ?

(4) Define a subgroup. State Lagrange's theorem. What can you say about the subgroups of a group having prime order ? Explain.

(5) Define a binary operation. What do you mean by closure property ? Is the set $\mathbb{R}$ of real numbers closed under addition ? Multiplication ? Division ? Explain. Is the set $\mathbb{Z}$ of integers closed under addition, multiplication or division ? Explain. Give an example of a set closed under some operation.