Seat No. : _____

DA-133

December-2013

5 Years M.Sc (CA & IT) Integrated (K.S.)

Sem-I (F.Y. M.Sc.)

CC-115 : MATHEMATICAL CONCEPTS

Time : 3 Hours] **[Max. Marks : 100**

1. (A) (a) The vertices of a triangle are (1, a) (2, b) and (c^2, –3), prove that its centroid cannot lie on the y-axis. **4**

(b) The co-ordinate of two points A and B are (3, 4) and (5, –2) respectively. Find the co-ordinates of any point P if PA = PB and are of ΔAPB is 10. **4**

(c) Prove that the points (a, b + c), (b, c + a) and (c, a + b) are collinear. **4**

OR

(a) Prove that in a right angled triangle the mid point of the hypotenuse is equidistant from its vertices.

(b) Find the co-ordinates of incentre of the triangle whose vertices are (4, –2), (-2,4) and (5,5).

(c) The vertices of a triangle are A(10, 4), B(– 4, 9) and C(–2, –1). Find the equation of the altitude through A.

(B) (a) The angle between two lines is $\frac{\pi}{4}$ and the slope of one of them is $\frac{1}{2}$. Find the slope of the other line. **4**

(b) If the straight line y = m*x* + c passes through the points (2, 4) and (–3, 6), find the values of m and c. **4**

OR

(a) A line passes through the points A(2, –3) and B(6, 3). Find the slopes of lines which are (i) parallel to AB line and (ii) perpendicular to AB line.

(b) For what value of k the points (k, 2 – 2k), (–k + 1, 2k) and (– 4 – k, 6 – 2k) are collinear ?

2. (A) (a) Prove that $\sin\frac{10\pi}{3}\cos\frac{11\pi}{6} + \cos\frac{2\pi}{3}\sin\frac{5\pi}{6} = -1$. **4**

(b) Prove that cos 10° + cos 50° + cos 70° + cos 110° + cos 130° + cos 170° = 0. **4**

(c) Prove that $\sin(45° + \theta)\sin(45° - \theta) = \frac{1}{2}\cos 2\theta$. **4**

OR

(a) Show that $\sin^2(51° + \theta) + \sin^2(39° - \theta) = 1$.

(b) If ΔABC is a right angled triangle then show that $\sin^2 A + \sin^2 B + \sin^2 C = 2$.

(c) Prove that $8\cos 20° \cos 40° \cos 80° = 1$.

(B) (a) Prove that $\frac{\sin 3\theta}{1 + 2\cos 2\theta} = \sin\theta$ and hence find the value of sin 15°. **4**

(b) Solve : $4\sin^2 2\theta - 3 = 0$. **4**

OR

(a) Express $\sin 5\theta$ in terms of $\sin\theta$.

(b) The angle of elevation of the top of a temple as observed from the foot of a tower is 60°, whereas the angle of elevation of the top of the tower as observed from the foot of the temple is 30°. If the tower is 50 m high, find the height of the temple.

3. (A) (a) Show that $\lim_{x \to 0} \frac{e^{1/x} - 1}{e^{1/x} + 1}$ does not exist. **4**

(b) If $f(x) = \begin{cases} 2x + 3, \text{ when } x < 0 \\ 0, \quad \text{when } x = 0 \\ x^2 + 3, \text{ when } x > 0 \end{cases}$ Discuss the continuity. **4**

(c) If $y = \sqrt{\cos x + \sqrt{\cos x + \sqrt{\cos x + \ldots \infty}}}$ then show that $\frac{dy}{dx} = \frac{\sin x}{1 - 2y}$. **4**

OR

(a) Evaluate $\lim_{x \to \infty} \left(x - \sqrt{x^2 + x}\right)$.

(b) If $y = e^{(\tan^{-1} x)^3}$, find $\frac{dy}{dx}$.

(c) If $y = x^{\sin x}$, find $\frac{dy}{dx}$.

(B) (a) If $x = a(t + \sin t)$ and $y = a(1 - \cos t)$, find $\frac{d^2y}{dx^2}$. **4**

(b) If $y = x^x$, prove that $\frac{d^2y}{dx^2} - \frac{1}{y}\left(\frac{dy}{dx}\right)^2 - \frac{y}{x} = 0$. **4**

OR

(a) If $y = e^x \tan x + x \log_e x$, find $\frac{dy}{dx}$.

(b) If $y = \sqrt{\frac{1 - x}{1 + x}}$, prove that $(1 - x)^2 \frac{dy}{dx} + y = 0$.

4. (A) (a) Evaluate : $\int \frac{1}{1 + \sin x}\, dx$. **4**

(b) Evaluate : $\int \frac{x^{e-1} + e^{x-1}}{x^e + e^x}\, dx$. **4**

(c) Evaluate : $\int x\sqrt{x^2 + a^2}\, dx$. **4**

OR

(a) Evaluate : $\int \frac{1}{\sqrt{x}} \cos\sqrt{x}\, dx$.

(b) Evaluate : $\int \frac{dx}{x[1 + (\log x)^2]}$.

(c) Evaluate : $\int \frac{1\text{-}\sin x}{x + \cos x}\, dx$.

(B) (a) Evaluate : $\int x \tan^2 x\, dx$. **4**

(b) Evaluate : $\int \frac{dx}{x - x^3}$. **4**

OR

(a) Evaluate : $\int \frac{xe^x}{(x+1)^2}\, dx$.

(b) Evaluate : $\int \frac{x^2}{(x-1)^2 (x^2+1)}\, dx$.

5. (A) (a) Determine the power series for $\tan x$ as far as the term in x^3. **4**

(b) Find the area of region bounded by $x = 3y^2 - 9$, y-axis and the lines $y = 0$ and $y = 1$. **4**

(c) Define order and degree of a differential equation with example. **4**

OR

(a) Expand $\ln(1 + x)$ to five terms.

(b) Find the area included between the curves $y^2 = 4ax$ and $x^2 = 4ay$.

(c) Determine the degree and the order of the following differential equations.

(i) $\left(\frac{d^3y}{dx^3}\right)^2 + 2\frac{d^2y}{dx^2} \cdot \frac{dy}{dx} + x^2\left(\frac{dy}{dx}\right)^3 = 0.$

(ii) $\sqrt{\frac{d^2y}{dx^2}} = 3.\frac{dy}{dx} + x.$

(B) (a) Find the general solution of the differential equation :

$2x\,(1 + y^2)dx - y(1 + 2x^2)\,dy = 0$ **4**

(b) Find the general solution of the differential equation : **4**

$x^2y\frac{dx}{dy} = x^3 + y^3.$

OR

(a) Find the general and particular solution of the differential equation :

$(e^y + 1)\cos x\,dx + e^y \sin x\,dy = 0$ and $x = \frac{\pi}{4} \Rightarrow y = 0.$

(b) Find the general solution of the differential equation :

$(x^2 - y^2)\,dx = 2xy\,dy$

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