Seat No. : _______________

DA-125

December-2013

B.Sc., Sem.-V

MAT-303 : Mathematics

(Complex Variables and Fourier Series)

Time : 3 Hours] **[Max. Marks : 70**

Instructions :
(1) **All** the questions are compulsory.
(2) Attempt any **two** from (a), (b) and (c) in the Q. **1** to Q. **4**
(3) Q. **5** is of short questions and it is compulsory.
(4) Each questions is of **14** marks.

1. (a) State atleast ten algebraic properties of the complex numbers. Show that $z = 1 \pm i$ satisfies the equation $z^2 - 2z + 2 = 0$. Also, find the four roots of the equation $z^4 + 4 = 0$; $z \in C$.

   (b) State the triangle inequality for the complex numbers, hence show that

   $\big|\,|z_1| - |z_2|\,\big| \le |z_1 - z_2|$; for $z_1, z_2 \in C$. Prove that $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\bar{z}_1}{\bar{z}_2}$, $z_2 \neq 0$, for $z_1, z_2 \in C$.

   (c) Is there any relation between the trigonometric and hyperbolic complex functions ? If yes, state it for all the trigonometric functions. Define convergence of sequence and series. Suppose that $z_n = x_n + iy_n$ $(n = 1, 2, ...)$ and $S = X + iY$ then prove that $\sum_{n=1}^{\infty} z_n = S$ if and only if $\sum_{n=1}^{\infty} x_n = X$ and $\sum_{n=1}^{\infty} y_n = Y$

2. (a) Derive Cauchy-Riemann equations $u_x = v_y$ and $u_y = -v_x$ stating necessary conditions and verify the same for the function $f(z) = e^z$, $z = x + iy$.

   (b) Show that the function $f(z) = \frac{x^3(1 + i) - y^3(1 - i)}{x^2 + y^2}$; $(x, y) \neq (0, 0)$

   $= 0$; $(x, y) = (0, 0)$

   is not analytic at $z = 0$ $(z = (x, y))$; even if $f(z)$ satisfies Cauchy-Riemann equations at origin i.e. $z = 0$.

   (c) Define harmonic function. Find the harmonic conjugate of $y^3 - 3x^2y$ and corresponding analytic function in terms of z.

3. (a) Find the image of strip $1 \le x \le 2$ under the mapping $w = \frac{1}{z}, z \neq 0$

(b) Prove : An analytic function f(z) is conformal at z_0 if and only if $f'(z_0) \neq 0$.

(c) Show that the angle between the curves $y = 2x$ and $y = x - 1$ is preserved under the mapping $w = z^2$.

4. (a) Define the Fourier series for the function f and obtain the same for the function $f(x) = x \sin x$, hence, deduce that $\frac{x}{4} = \frac{1}{2} + \frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - \ldots\ldots\ldots$

(b) State and prove Bessel's inequality.

(c) Find the Fourier series expansion of the function $f(x) = x + x^2$ in $[-\pi, \pi]$.

5. (i) Simplify : $\left(\frac{1}{2 - 3i}\right)\left(\frac{1}{1 + i}\right)$

(ii) Write the C-R equations and derivative of the complex function f(z) in polar form.

(iii) Is the function $f(z) = |z|^2$ analytic ? Justify.

(iv) Find the singular points of $|z|^2$ and $\frac{1}{z}$.

(v) Find the non-conformal points of the mapping $f(z) = 2z^3 + 15z^2 - 6z + 9$.

(vi) Obtain $\int_{-\pi}^{\pi} \cos nx \, dx$ for all n.

(vii) Obtain $\int_{-\pi}^{\pi} \sin mx \cos nx \, dx$ for all m, n = 0, 1, 2, ………