Seat No. : _____

LG-109

April-2014

B.Sc. (Sem.-VI)

CC-310 : Mathematics

(Graph Theory)

Time : 3 Hours] **[Max. Marks : 70**

Instructions : (1) **All** the questions are compulsory and carry **14** marks.
(2) Figures to the right indicate marks of the question.

1. (a) If G is any graph with e edges and n vertices $v_1, v_2, ..., v_n$ then prove that $\sum_{i=1}^{n} d(vi) = 2e$. Also prove that G must have even number of odd vertices. **7**

OR

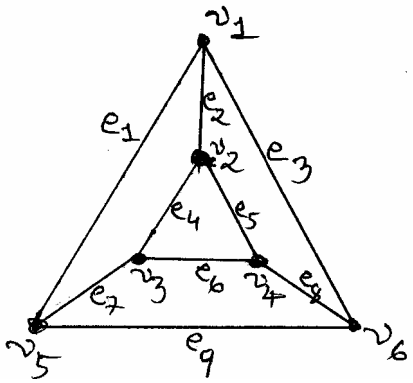
Define isomorphism of graphs and give one example of isomorphic graphs and one example of non-isomorphic graphs.

(b) Given any two vertices u and v of a graph G. Prove that every u-v walk contains a u-v path in a graph G. **7**

OR

Define the k-cube Q_k for integer $k \geq 1$. Show that Q_k has 2^k vertices and $k.2^{k-1}$ edges. Also show that Q_k is bipartite.

2. (a) Write down the adjacency and incidence matrices of the following graph : **7**



OR

(a) Prove that an edge '*e*' of a graph G is a bridge if and only if 'e' is not a part of any cycle in the graph G.

(b) If *u* and *v* are distinct vertices of a tree T, then prove that there is precisely one path from *u* to *v*. **7**

OR

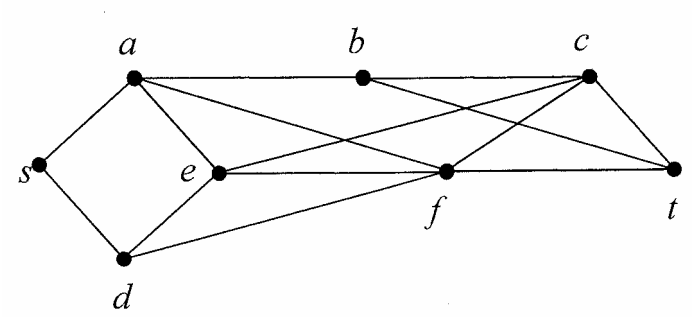
Prove that a connected graph G is a tree if and only if every edge of G is a bridge.

3. (a) If G is a connected graph, then prove that it has a spanning tree. **7**

OR

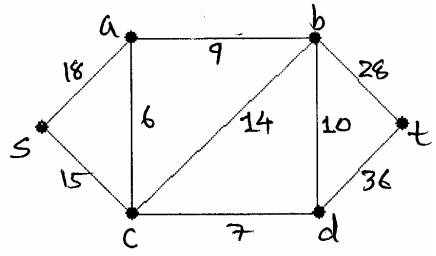
Prove that a vertex *v* of a connected graph G is a cut vertex of G if and only if there are two vertices *u* and *w* of G different from *v* such that *v* is on every *u*-*w* path in G.

(b) Explain the Breadth First Search algorithm and find the shortest path from '*s*' to '*t*' in the following graph : **7**



OR

Apply the Dijkstra's algorithm on the following connected weighted graph to find the length of shortest paths from the vertex s to each of the other vertices :



4. (a) If G is a graph in which the degree of every vertex is at least two, then prove that G contains a cycle. **7**

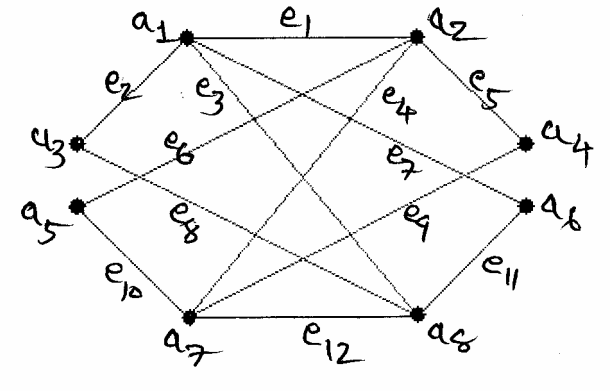
OR

State and prove the Dirac's theorem for a Hamiltonian graph.

(b) Prove that a connected graph G is Euler if and only if every vertex of G is of even degree. **7**

OR

(b) Use the Fleury's algorithm to produce an Euler tour for the following graph :



5. Attempt any **seven** of the followings in short : **14**

(a) Define any two : (i) Loop (ii) Parallel edges (iii) Simple Graph

(b) Define any two : (i) Bipartite Graph (ii) Complete Graph (iii) Complete Bipartite Graph.

(c) If $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ is the adjacency matrix of the graph G, then draw the graph G.

(d) Explain fusion of vertices in any connected graph.

(e) Define any two : (i) Tree (ii) Bridge (iii) Cut vertex.

(f) Draw a graph representing the bridges of Konigsberg.

(g) State the Caley's theorem for the Complete Graph.

(h) Define a minimal spanning tree.

(i) Define a maximal non-Hamiltonian Graph.

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