Seat No. : _______________

LF-110

April-2014

B.Sc. Sem.-VI

CC – 309 : Mathematics

(Analysis – III)

Time : 3 Hours] **[Max. Marks : 70**

Instructions : (1) **All** questions are compulsory.

(2) Write the question number in your answer sheet as shown in the question paper.

(3) Figures to the right indicate marks of the question.

1. (a) Prove that each closed sphere is a closed set in any metric space X. **7**

OR

Let X be a complete metric space and let $\{f_n\}$ be a decreasing sequence of non-empty subsets of X such that $d\{f_n\} \to 0$. Then prove that $F = \bigcap_{n=1}^{\infty} f_n$ contains exactly one point.

(b) Prove that BdryA is closed set. **7**

OR

Let $x_n = \frac{1}{n}$, $n \in N$ then show that $\{x_n\}$ is convergent. Also find its limit point.

2. (a) Let x and y be two metric spaces and f be a mapping of x into y ; then prove that f is continuous if and only if $f^{-1}(G)$ is open in x whenever G is open in y. **7**

OR

The metric space (x, d) is compact if and only if every sequence of points in x has a subsequence converging to a point in x.

(b) If f is a continuous real valued function on the closed and bounded interval [a, b] then prove that f takes every value between f(a) and f(b). **7**

OR

Prove : The function $f : (0, 1) \to R$ defined by $f(x) = \frac{1}{x}$ is not uniformly continuous.

3. (a) Let (f_n) be a sequence of continuous function on $E \subset C$ converging uniformly to f on E, then prove that f is continuous on E. Justify the converse of this theorem. **7**

OR

Let f_n satisfy

(i) $f_n \in D[a, b]$

(ii) $f_n(x_0)$ converges for some $x_0 \in [a, b]$

(iii) f'_n converges uniformly on [a, b] then prove that (a) f_n converges uniformly on [a, b] to a function f and (b) $f'(x) = \lim_{n \to \infty} f_n'(x)$.

(b) To give an example to show that $\lim_{n \to \infty} \int f_n(x)dx \neq \lim_{n \to \infty} \int f_n(x)dx$ **7**

OR

Show that $f_n(x) = n^2 x^n (1 - x)$, $x \in [0, 1]$ converges pointwise but not uniformly to a function which is continuous on [0, 1].

4. (a) Let $f(x) = \sum a_n x^n$ be a power series with radius of convergence 1, if the series converges at 1, then prove that $\lim_{x \to 1} f(x) = f(1)$. **7**

OR

Let $f : [0, 1] \to R$ be continuous. Then prove that there exists a sequence of polynomial the Bernstein polynomials $B_n f$ such that $B_n f \to f$ uniformly on [0, 1].

(b) Show that for $-1 < x < 1$ **7**

$\log(1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} \cdots$ and $\log(2) = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots$

OR

Discuss the radius of convergence of following series : **7**

(i) $\sum_{n=0}^{\infty} \frac{x^n}{n^2}$

(ii) $\sum_{n=1}^{\infty} \frac{x^{n-1}}{n}$

5. Give the answer in short : (Any **seven**) **14**

(1) Define open sphere.

(2) Give an example to show that the arbitrary intersection of open sets is not open.

(3) Find the interior of

(i) The set of all integers I

(ii) The set of all real number R

(4) Find the derived set of

(i) A = (2, 3)

(ii) A= $\{z \mid z \mid < 1\}$

(5) Prove that [0, 1] is compact.

(6) Define uniformly continuous.

(7) Say : The metric space X = R – {0} is connected or disconnected.

(8) For which value of x the geometric series $\sum_{n=0}^{\infty} x^n$ is convergent ?

(9) For which value of α the series $s(z) = \sum_{n=1}^{\infty} \frac{z^n}{n^\alpha}$ is uniformly continuous on [–1, 1]