Seat No. : _______________

LE-110

April-2014

B.Sc. Semester – VI

CC-308 : Mathematics (Analysis – II)

Time : 3 Hours] **[Max. Marks : 70**

Instructions : (i) **All** the **five** questions are compulsory.

(ii) Each question is of **14** marks.

(iii) Figures to the right indicate marks of the question.

1. (a) Define : Riemann integrable function on [a, b]. Let a function $f(x) = \frac{3x^2}{2}$ and $P = \left\{0, \frac{1}{n}, \frac{2}{n}, \frac{3}{n}, ..., \frac{n}{n}\right\}$ be a partition of [0, 1], then compute $\lim_{n \to \infty} U[f; P]$ and $\lim_{n \to \infty} L[f; P]$. **7**

OR

Prove : If f, g $\in$ R [a, b] and g is bounded away from zero, then fg $\in$ R [a, b] and $\frac{f}{g} \in R[a, b]$.

(b) Let g be continuous on [a, b] and f has derivative which is continuous and never changes sign then for some a $\le$ c $\le$ b prove that $\int_a^b f(x)\, g(x)\, dx = f(a) \int_a^c g(x)\, dx + f(b) \int_c^b g(x)\, dx$. **7**

OR

Examine the validity of the expression $\frac{2\pi^2}{9} \le \int_{\pi/6}^{\pi/2} \frac{2x\, dx}{\sin x} \le \frac{4\pi^2}{9}$. Is the statement $|f| \in R[a, b] \Rightarrow f \in R[a, b]$ true ? Explain.

2. (a) Define convergent series and show that the limit of the n-th term of the convergent series is zero. Discuss the convergence of the series $\sum_{n=1}^{\infty} \frac{1^2 + 2^2 + 3^2 + ... + n^2}{5n^5 + 2n^2 + 9}$ **7**

OR

State and prove Cauchy's condensation test and hence, show that the series $\sum_{n=2}^{\infty} \frac{1}{n(\log n)^p}$ converges for $p > 1$ and it diverges for $p \le 1$.

(b) Define absolute convergence of the series. Discuss the absolute convergence of the series $\sum_{n=1}^{\infty} (-1)^{n+1} \left(\frac{\sqrt{n^2 + 7} - n}{\sqrt{n}}\right)$. **7**

OR

Define an alternating series. If $a_{n+1} \le a_n$ and $\lim_{n \to \infty} a_n = 0$, then prove that alternating series $\sum_{n=1}^{\infty} (-1)^{n+1} a_n$ converges. Discuss the convergence of the series $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n \cdot (n + 1)}$.

3. (a) Prove : If $\sum a_n$ is absolutely convergent then any rearrangement of $\sum a_n$ converges to the same sum. **7**

OR

State and prove Mertens' theorem for the Cauchy product of two series.

(b) Define power series centered at x_0. Find the radius of convergence of the following power series : **7**

(i) $\sum \frac{10^n x^n}{2^{n^2}}$

(ii) $\sum \frac{3^n x^n}{n!}$

OR

Define improper integral of first and second kind. Test convergence :

(i) $\int_0^{\infty} \frac{dx}{e^x}$ (ii) $\int_{-\infty}^{\infty} \frac{dx}{1 + x^2}$

4. (a) Define Taylor's series for the function f about x_0. State Taylor's theorem. Using Lagrangian form for the remainder, for any real x show that $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + ...$ **7**

OR

For $-1 \leq x < 1$, show that $\ln(1 - x) = -x = \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - ....$ hence, deduce that $\ln 2 = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + ...$

(b) Obtain the power series solution of the differential equation $(1 - x)\, y' + 1 = 0$ with the condition $y(0) = 1$. **7**

OR

Show that $(1 + x)^{\alpha} \approx 1 + \sum_{n=1}^{\infty} \frac{\alpha(\alpha - 1)(\alpha - 2) .... (\alpha - n + 1)}{n!} x^n$ and find its radius of convergence. Give suitable name to this result.

5. Attempt any **seven** in short : **14**

(i) State first fundamental theorem for the R-integrable function.

(ii) If f R-integrable function then show that $\left| \int_a^b f(x)\, dx \right| \leq \int_a^b |f(x)|\, dx$

(iii) For the series Σa_n if $\lim_{n \to \infty} a_n \neq 0$ then what can be said about the convergence of Σa_n ? Justify.

(iv) Discuss the convergence of $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$. Does it converge absolutely ? Justify.

(v) Test the convergence of $\int_{-\infty}^{1} \frac{1}{2^{x-1}}\, dx$.

(vi) Find the Cauchy product of the series $\sum_{n=0}^{\infty} \frac{(-1)^n}{n + 1}$ with itself.

(vii) State the series of e^x, for any real x.

(viii) State the Binomial series theorem.

(ix) If $y(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + ...$ and $y(x) = y'(x)$, then find the coefficient $a_1, a_2, a_3, \ldots, a_n$ in terms of a_0.
