Seat No. : _______________

XZ-129

April-2013

B.Sc. (Sem.-IV)

Mathematics : 204

(Advance Calculus-II)

Time : 3 Hours] **[Max. Marks : 70**

Instructions : (1) **All** the questions are compulsory.

(2) Each question carry **14** marks.

(3) Notations are usual.

1. (a) Change the order of integration in the integral $\int_0^a \int_{x^2/a}^{2a-x} xy \, dy \, dx$ and hence evaluate it.

OR

Change the order of integration in the integral $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y} \, dy \, dx$ and hence evaluate it.

(b) Evaluate : $\int_0^3 \int_0^{\sqrt{9-x^2}} xy \, dx \, dy$

OR

Evaluate : $\int_1^e \int_1^{\log y} \int_1^{e^x} \log z \, dz \, dx \, dy$

2. (a) State and prove Duplication formula for beta and gamma function.

OR

Define Divergence of a vector function in R^3. Prove that

$\mathrm{div}(\bar{f} \times \bar{g}) = \bar{g} \cdot \mathrm{curl}\ \bar{f} - \bar{f} \cdot \mathrm{curl}\ \bar{g}$

(b) (i) Prove that $\mathrm{div}(r^n \bar{r}) = (n + 3)r^n$

(ii) Prove that $\nabla^2 (r^n \bar{r}) = n(n + 3)r^{n-2}\, \bar{r}$

OR

Prove that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$

3. (a) State and prove Green's theorem.

OR

State and prove Stoke's theorem.

(b) Using Green's theorem evaluate $\oint_C (3x^2 - 8y^2)dx + (4y - 6xy)dy$, where C is the boundary of the region bounded by $Y^2 = X$ and $X^2 = y$.

OR

Verify Gauss Divergence theorem for the vector field $\overline{F}$ on the region V, where $\overline{F}(X, Y, Z) = XY\overline{i} + YZ\overline{j} + ZX\,\overline{k}$, V is the solid cylinder $X^2 + Y^2 \leq 1$, $0 \leq Z \leq 1$.

4. Attempt any **two** :

(a) Define Partial Differential Equation. State Lagrange's equation for P.D.E and discuss the method for solving it.

(b) Solve P.D.E. $x^3p + y^3q = (x^2 - xy + y^2)z$.

(c) Derive P.D.E. for $f(X - Z, Y - Z) = 0$.

5. Answer in short :

(a) If $B(x, 2) = \frac{1}{3}$, then find the value of x.

(b) Show that $B(m + 1, n) = B(m, n + 1) = B(m, n)$

(c) If $\phi = XYZ$, then find the value of $|\text{grad}\phi|$ at the point (1, 2, –1).

(d) If $\overline{r} = X\overline{i} + Y\overline{j} + Z\overline{k}$, then find div $\overline{r}$.

(e) Evaluate $\int (xdy - ydx)$ over the parabola $y = x^2$ from (0, 0) to (1, 1).

(f) Obtain the area of region R by Green's theorem.

(g) Find P.D.E. of $z = y + ax^2\,y + b$, where a and b are parameters.