Seat No. : _______________

TB-117

April-2013

M.Sc. Sem. IV Examination

509EA : Mathematics

(Mathematical Methods)

Time : 3 Hours] **[Max. Marks : 70**

1. (a) Attempt any **one** : **7**

   (i) Solve : $4xy'' + 2y' + y = 0$.

   (ii) Find the eigen values and eigen functions of the problem

   $$y'' + \lambda y = 0,\ y(0) = 0,\ y'(1) = 0.$$

   (b) Attempt any **two** : **4**

   (i) Solve $y' = 2xy$ in terms of a power series in x.

   (ii) Using the indicated substitutions, reduce the following equation to Bessel's differential equation

   $$y'' + x^2y = 0, \left(y = u\sqrt{x},\ \frac{1}{2}x^2 = z\right)$$

   (iii) Show that $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \sin x$.

   (c) Answer very briefly. **3**

   (i) Find the indicial equation and its roots of the equation

   $$xy'' + 2y' + 4xy = 0.$$

   (ii) Show that $J_{\nu-1}(x)\ J_{\nu+1}(x) = \frac{2\nu}{x} J_\nu(x)$.

   (iii) What is Sturm-Liouville problem ?

2. (a) Attempt any **one**. **7**

   (i) Using the Laplace transform, solve the following initial value problem.

   $$y'' + 6y' + 8y = e^{-3t} - e^{-5t},\ y(0) = 0,\ y'(0) = 0$$

   (ii) State and prove first shifting theorem. Using first shifting theorem, obtain the Laplace transform of $5e^{2t} \sinh 2t$.

(b) Attempt any **two** : **4**

(i) Find the Laplace transform of $4u(t-\pi)\cos t$.

(ii) Find the inverse Laplace transform of $\frac{e^{-3s}}{(s-1)^3}$.

(iii) Using the convolution, find the inverse Laplace transform of $\frac{1}{s^2(s-1)}$.

(c) Answer very briefly. **3**

(i) Find the Laplace transform of te^t.

(ii) State second shifting theorem.

(iii) Define Dirac delta function.

3. (a) Attempt any **one**. **7**

(i) Find the Fourier series of

$$f(x) = \begin{cases} x & \text{if } -\frac{\pi}{2} < x < \frac{\pi}{2} \\ \pi - x & \text{if } \frac{\pi}{2} < x < \frac{3\pi}{2} \end{cases}$$

(ii) Show that

$$\int_0^\infty \frac{\cos xw + w \sin xw}{1+w^2}\, dw = \begin{cases} 0 & \text{if } x < 0 \\ \frac{\pi}{2} & \text{if } x = 0 \\ \pi e^{-x} & \text{if } x > 0 \end{cases}$$

(b) Attempt any **two**. **4**

(i) Find the Fourier cosine integral of $f(x) = e^{-kx}$ $(x > 0, k > 0)$.

(ii) Find $F_s(e^{-ax})$, $a > 0$, by integration.

(iii) Using Parseval's identity, prove that

$$1 + \frac{1}{2^4} + \frac{1}{3^4} + \frac{1}{4^4} + \dots = \frac{\pi^4}{90}.$$

(c) Answer any briefly. **3**

(i) Show that the Fourier transform is a linear operation.

(ii) Define total square error.

(iii) What is the smallest positive period p of $\sin \frac{2\pi n x}{k}$?

4. (a) Attempt any **one**. **7**

(i) Find the Z-transforms of $\{a^{|k|})$ and $\left\{\frac{1}{3^k}\right\}$.

(ii) Find the inverse Z-transform of

$$\frac{2z^2 - 10z + 13}{(z-3)^2 (z-2)},$$

when $2 < |z| < 3$.

(b) Attempt any **two** : **4**

(i) Prove that if $Z[\{f(k)\}] = F(z)$, then $Z[\{a^k f(k)\}] = F\left(\frac{z}{a}\right)$

(ii) Find the Z-transform of $c^k \sin \alpha k$, $k \geq 0$.

(iii) Find the inverse Z-transform of $\frac{z}{(z-1)(z-2)}$ by residue method.

(c) Answer very briefly. **3**

(i) State initial value theorem.

(ii) Find z-transform of $\left(\frac{1}{2^k}\right)$, $-4 \leq k \leq 4$

(iii) State final value theorem.

5. (a) Attempt any **one** : **7**

(i) Prove $H_n\left\{\frac{df}{dx}\right\} = -s\left[\frac{n+1}{2n} H_{n-1}\{f(x)\} - \frac{n-1}{2n} H_{n+1}\{f(x)\}\right]$

(ii) Show that

$$\int_0^a x^3 J_0(sx)dx = \frac{a^2}{s^2}\left[2J_0(as) + \left(as - \frac{4}{as}\right)J_1(as)\right].$$

(b) Attempt any **two** : **4**

(i) Show that $H\{f(ax)\} = a^{-2}H\left[\frac{s}{a}\right]$.

(ii) Find the Hankel transform of zero order of $\frac{\sin ax}{x}$.

(iii) Find $H^{-1}\,[s^{-2}\,e^{-as}]$, when n = 1.

(c) Answer very briefly. **3**

(i) Evaluate $\int_0^a x^2 J_1(sx)dx$.

(ii) Show that Hankel transform is a linear operation.

(iii) Find $H[e^{-ax}]$, n = 0.