Seat No. : _______________

# XW-113

**April-2013**

## M.Sc. (Sem.-II)

## 407 - STATISTICS

## (Reliability and Life Testing and Bayes Estimation)

**Time : 3 Hours]** **[Max. Marks : 70**

**Instructions :** (1) **All** questions carry equal marks.
(2) Scientific calculator is permitted to use.
(3) Statistical table will be supplied on request.

1. (a) Define reliability and hazard function of an device. Obtain expression for h(t). The CDF of the lifetime (in months) of an electronic device is $F(t) = t^3/216$, if $0 \le t \le 6$; otherwise 1.
(i) What is the hazard function of this device ?
(ii) What is mtbf ?

**OR**

What do you mean by life of a device and life testing ? Describe a bathtub failure curve. Mention some situations where such a curve would provide appropriate description.

(b) Define cumulative hazard function $H(t, \theta)$. Prove that
$$R(t) = e^{-H(t,\, \theta)}$$
If reliability of an item is $\exp(-((t - \mu)/\theta)^{\beta})$, obtain its hazard and cumulative hazard function.

**OR**

If hazard function of an item is $\lambda(t) = \frac{1}{\sigma} \exp\left(\frac{t-\mu}{\sigma}\right)$, $-\infty < t < \infty$ obtain (i) reliability function (ii) pdf of life time distribution of the component (iii) If $\mu = 0$, $\sigma = 1$, obtain probability that the device will not fail before 2 time unit.

2. (a) Consider a system of two independent components each having constant failure rate $\theta_1 > 0$ and $\theta_2 > 0$ respectively. Show that if the system is series system then expected life of the system is less than the expected life of either component but if the system is parallel then it is more than that of either component.

**OR**

Obtain hazard rate of the series and parallel system having two independent components having constant equal hazard rate.

(b) If h(t) be the hazard rate of a device having continuous life time X. Show that

(i) E(X-c I) = E(X)

(ii) $$\frac{P(X > b)}{P(X > a)} = \exp\left[-\int_a^b h(t)\, dt\right]$$

**OR**

Let $X_1, X_2, \ldots, X_{10}$ be the life time random variables of a random sample of size n =10 devices having identical exponential life time model with mean life time of 1000 hrs.

(i) What is expected time till to first failure ?

(ii) Let $X_{(1)} \leq X_{(2)} \leq \ldots \leq_{(10)}$ be the order statistics of this sample. Are $X_{(6)} - X_{(5)}$ and $X_{(5)} - X_{(4)}$ independent ? Prove your answer.

3. (a) What is type II censoring ? Obtain MLE of θ under type II censoring without replacement in case of exponential life time model with mean θ, θ > 0 based on a random sample of size n. Also obtain UMVUE of R(T).

**OR**

Under type I censoring without replacement, obtain MLE of the mean life time of exponential life time model based on sample size and number of failures only. Also obtain asymptotic variance of the estimator.

(b) In case of life time model $f(x; \mu, \theta) = \frac{1}{\theta} e^{-(x - \mu)/\theta}$, $x > \mu$, $\theta > 0$ obtain MLE of θ and μ under type II censoring without replacement based on a sample of size n. Also deduce UMVUE of θ and μ.

**OR**

Discuss Sinha and Fue method of estimation of failure rate of Weibull lifetime model. Hence how do you get least square estimates of the parameters of the model ?

4. (a) Define risk function and Bayes risk. Obtain general form of the Bayes estimator of g(θ), a function of θ under weighted squared error loss function.

**OR**

Show that the Bayes estimator which minimize the Bayes risk can also be obtained by minimizing the posterior expected loss.

(b) Let $X_1, X_2, \ldots, X_n$ be a random sample of size n from Bernoulli distribution with parameter p, $0 < p < 1$ and the prior distribution of p is $\beta_{(l,\ m)}$ beta type I distribution. Obtain Bayes estimator of (i) $p(1-p)$ and (ii) $p^2$ under weighted square error loss function with weight $w(p) = p(1-p)$. Also find corresponding Bayes risk.

**OR**

Let $X_1, X_2, \ldots, X_n$ be a random sample of size n from $U(0, \theta)$ uniform distribution. Let prior distribution of $\theta$ is $\pi(\theta) = \beta a^{\beta-1}/\theta^{\beta+1}$; $a < \theta < \infty$. Obtain Bayes estimate of $\theta$ under squared error loss function. Is it unbiased ? Verify your answer.

5. (i) State the type of censoring characterizes in the following study :
A study of television failures is going on to be conducted during the next three years. The television enter the study as they are sold. Assume that the sale times are randomly distributed over the study period. If a customer leaves the area then her/his set up drops from the study.

(ii) A system having two components C1 and C2 connected in parallel. Suppose that life time of C1 has exponential distribution with mean $\theta$ and the life time of C2 has Rayleigh distribution with hazard rate $x/\theta^2$. What will be the system reliability ?

(iii) Say True or False.
Hazard rate of a series system having two components is equal to the sum of the hazard rate of its components.

(iv) Say True or False.
When failure rate is constant, then mtbf is the reciprocal of the constant failure rate.

(v) According to Fisher, the estimate of hazard function is ______.

(vi) If R(T) of a device is $\frac{1}{2} e^{-t/2} + \frac{1}{2} e^{-t/3}$ then its mtbf is ______.

(vii) If observed total test time of an experiment with a sample of size 10 when the test is terminated at third failure is 327 hrs then MLE of mean life time of exponential life time model is ______.

(viii) Say True or False.
MLE of mean life time $\theta$ of exponential distribution under type II censoring without replacement and with replacement are equally precise.

(ix) If ten items are placed on a test and when no failure observed during the fixed test time interval (0, 30) hrs then MLE of mean life time under exponential life time model is _______.

(x) State UMVUE of R(t) under type-I censoring with replacement based on sample size and number of failures only.

(xi) Let X ~ Poisson distribution with mean θ, $\theta > 0$. Then appropriate prior for parameter θ is

(a) Gamma G(p), $p > 0$

(b) Normal $N(0, \sigma^2)$, $\sigma > 0$

(c) Extreme value distribution

(d) Uniform U(–a, a), $a > 0$

(xii) Let $X \sim N(\theta, \sigma^2)$, $\sigma > 0$ is known. The appropriate prior distribution of θ is

(a) exponential with mean β, $\beta > 0$.

(b) N(μ, σ2)

(c) Uniform U(–1, 1)

(d) Beta type – II distribution

(xiii) Say True or False.

The posterior distribution is updated version of the prior distribution of parameter θ, hence it has always more precision than the prior distribution.

(xiv) Which one is the informative prior for parameter θ in case of exponential distribution with Mean 1/θ, $\theta > 0$ ?

(α) εξπ (μεαν 1/β)

(b) U(0, 1)

(c) exp (mean 1)

(d) N (0, 1)

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