Seat No. : _______________

XD-129

T.Y.B.Sc.

March-2013

Mathematics : Paper – X (Optional)

(Number Theory & Combinatorics)

Time : 3 Hours] **[Max. Marks : 105**

Instructions : (1) **All** questions are compulsory.

(2) Each question carries equal marks. Figures to the right indicate the maximum marks.

1. (a) Attempt any **two :** **15**

(1) State and prove the division algorithm for two integers.

(2) Solve in Z : $56x + 72y = 40$

(3) If $p \geq q \geq 5$, p & q are both primes then prove that $24 \mid p^2 - q^2$

(b) Give the answer of following : **6**

(1) State the fundamental theorem of arithmetic.

(2) Evaluate : $\gcd(2a + 1, 9a + 4)$

(3) Is 209 prime ? (Verify by Sieve of Eratosthenes)

2. (a) Attempt any **two :** **15**

(1) State and prove the Fermat's little theorem. Is converse true ? Justify.

(2) Solve : $17x \equiv 9 \pmod{276}$

(3) If 792 divides the integer $13xy\,45z$ then find the digits x, y and z.

(b) Give the answer of following : **6**

(1) Is 1010908899 divisible by 7, 11 and 13 ? Justify.

(2) Find the remainder when $\sum_{n=1}^{10} (n!)$ is divided by 20 ?

(3) State the Wilson's theorem.

3. (a) Attempt any **two :** **15**

(1) Define : $\tau(n)$ & $\sigma(n)$, $n \in N$ and prove that τ & σ are multiplicative functions.

(2) State and prove the Euler's Theorem.

(3) Prove : For any odd integer a, $a^{33} \equiv a \pmod{4080}$

(b) Give the answer of following : **6**

(1) Define : $\mu(n)$, $n \in N$ and find $\mu(210)$

(2) Find $\tau(13608)$ & $\sigma(13608)$

(3) Find last two digit of 7^{89}

4. (a) Attempt any **two :** **15**

(1) Solve : $x^2 + 7x + 10 \equiv 0$ (mo11)

(2) Find three roots of show that $4x^9 \equiv 7(\text{mod } 13)$ has three distinct roots

(3) Prove : $\left(\frac{2}{p}\right) = \begin{cases} +1 & , p \equiv \pm 1(\text{mod } 8) \\ -1 & , p \equiv \pm 3(\text{mod } 8) \end{cases}$

(b) Give the answer of following : **6**

(1) Define : Primitive root of integer n > 1. Write a least positive primitive root of 41.

(2) Define the order of an integer modulo n.

(3) Find the value of Legendre symbol : $\left(\frac{773}{461}\right)$, where 461 is a prime.

5. (a) Attempt any **two :** **15**

(1) Prove : $\binom{2n+1}{0} + \binom{2n+1}{1} + \binom{2n+1}{2} + \ldots + \binom{2n+1}{n} = 2^{2n}$

(2) $\binom{2p}{p} \equiv 2(\text{mo } p), \forall$ odd prime p

(3) There are 7 boys & 7 girls in a gathering. In how many ways can they be arranged on a round table so that particular boy and girl are not adjacent ?

(b) Give the answer of following : **6**

(1) Define : AP & MP.

(2) Find the number of permutations from the letters of the word "COLLEGE".

(3) State the Vandermonde's Identity.

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Mathematics : Paper – X (Optional)

Mechanics

Time : 3 Hours] **[Max. Marks : 105**

Instructions : (1) **All** questions are compulsory.
(2) Give your answers in usual notations, (if necessary).
(3) Write question number and subquestion number in answer sheet according to the question paper.

1. (a) Define level curve. State and prove Lami's theorem. **7**

OR

Define circumcentre of a triangle. State and prove theorem of triangle of forces and its converse.

(b) Attempt any **two :** **14**

(1) If O is the circumcentre of a triangle, forces P, Q, R act in the directions of OA, OB, OC and particle is in equilibrium, then show that

$$\frac{P}{a^2 (b^2 + c^2 - a^2)} = \frac{Q}{b^2(c^2 + a^2 - b^2)} = \frac{R}{c^2 (a^2 + b^2 - c^2)} .$$

(2) A particle of weight W is suspended from a fixed point by a light string. A horizontal force is applied to it and the particle takes up the position of equilibrium with the string inclined to the vertical. If the string breaks when the tension in it reaches a value T_0, find the smallest value of H necessary to break the string.

(3) P and Q are maximum and minimum resultants of two forces. If these two forces make an angle α with each other and their resultant is R then show that $R^2 = P^2 \cos^2 (\alpha/2) + Q^2 \sin^2(\alpha/2)$.

2. (a) Obtain the Cartesian equation of a curve formed by a uniform cable hanging freely under its own weight. **7**

OR

In usual notations, prove for catenary

(1) $y = c \sec\psi$

(2) $y^2 = s^2 + c^2$

(3) $x = c \log [(s + y) /c]$

(b) Attempt any **two :** **14**

(1) Discuss the centre of gravity of a solid right circular cone and hollow right circular cone with the height h.

(2) A uniform cable hangs across two smooth pegs at the same height the ends hanging down vertically. If the free ends are each 12 ft. long and the tangent to the catenary at each peg makes an angle 60° to horizontal, find the total length of the cable.

(3) In usual notations prove that $N = -(T / \rho)$.

3\. (a) Define kinetic energy. State and prove Koning's theorem. **7**

OR

Define moment of inertia. Discuss the motion of a solid body with respect to its centre of mass.

(b) Attempt any **two :** **14**

(1) A particle moving with a speed of 30 ft/sec in a direction making an angle 60° with the horizontal plane and rebounds, the coefficient of restitution being (1/3). Find the speed and direction of motion of the particle immediately after impact.

(2) Two balls of equal mass are at rest in one line. One of this ball is thrown on the other with the velocity u. Show that because of collision loss of kinetic energy is $\frac{(1 - e^2)\, mu^2}{4}$.

(3) State and prove theorem of parallel axes and perpendicular axes.

4\. (a) Define rehonomic motion. Derive the equation of motion for a simple pendulum of length 1 and mass of the body m by using Lagrange's equations. **07**

OR

Define holonomic motion. Prove for Hamilton's function that $\frac{dH}{dt} = \frac{\partial H}{\partial t}$. **07**

(b) Attempt any **two :** **14**

(1) If particle of mass m is moving in a plane under an attractive force $\left(\frac{\mu m}{r}\right)$ directed towards the origin of polar coordinates (r, θ). Find Lagrangian equations.

(2) Derive Lagrange's equations for impulsive motion.

(3) Find the differential equation for a compound pendulum which oscillates in a vertical plane about fixed horizontal axes by using Lagrange's equations.

5\. Attempt any **three :** **21**

(1) Derive equation of motion in relative form.

(2) In usual notations discuss the immediate consequence of Lorentz transformation on (i) contraction of length of a rod and (ii) time dialation.

(3) For accelerated motion of an object in the theory of relativity, show that $T' < T$.

(4) The length of a space vehicle is 100 m on ground. When it is in flight, its length as observed on the ground is 99 m. Calculate its speed.

(5) Find the velocity at which the mass of a moving body is double its rest.

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March-2013

Mathematics : Paper – X (Optional)

(C Programming & Algorithms)

Time : 3 Hours] **[Max. Marks : 70**

Instructions : (1) There are four questions. **All** questions are compulsory.

(2) Figures to the right indicate full marks for the questions.

1. (a) Explain the arithmetic operators with suitable examples. Which arithmetic operator is different in integer arithmetic and in real arithmetic ? What is the hierarchy among the arithmetic operators ? **7**

OR

Write a program to determine and print the roots of a quadratic equation

$ax^2 + bx + c = 0, a \in R, a \neq 0, b \in R, c \in R.$

(b) Determine the output of the following program : **6**

(i)

```
# inclue <stdio.h>
{
int i = 2012;
printf(”i=%d”, + + i);
printf(”i2=%d”, i – –);
printf(”i3=%d”,i + +);
printf(”i4=%d”, – –i);
}
```

(ii)

```
# include <stdio.h>
{
int a = 2013, b = 2012;
if (a > b)
{  a – = 1;  b + = 1; }
else
{  a + = 1; b – = 1;  }
printf(”a = %d, b = %d”’, a, b);
}
```

(c) State whether the following statements are true or false. **5**

(i) A line in a program have exactly one one statement.

(ii) Use of comments can not reduce the speed of execution of a program.

(iii) The underscore can be used anywhere in an identifier.

(iv) All arithmetic operators does not have the same level of precedence.

(v) The operators < =, > = and ! = all enjoy the same level of priority.

OR

Fill in the blanks with appropriate words : **5**

(i) The __________ function is used to display the output on the screen.

(ii) The _________ header file contains mathematical functions.

(iii) The escape sequence character ___________ causes the cursor to move to the next line on the screen.

(iv) ____________ is used to determine the order in which different operators in an expression are evaluated.

(v) Increment and decrement operators can be used with only ___________ type of variables.

2. (a) Explain the various **if** statements and the **switch** statements. Explain the differences between **if …. else** and **switch** statements with suitable examples. **7**

OR

Explain the differences between **while** and **do… while** statements with suitable examples.

(b) Write a **single** program which reads the value of a real variable *x* and evaluate y, where :

$$y = \begin{cases} 2012 + x & \text{if } x < 0; \\ x - 2013 & \text{if } x = 0; \\ x - 2014 & \text{if } x < 0 \end{cases}$$

using (i) netsted **if** statement, (ii) **else if** statement, (ii) Conditional Operator ? **6**

OR

(c) State whether the following statements are true or false with proper justification. **5**

(i) The **scanf** function cannot be used to read a single character from the keyboard.

(ii) The print list in a **printf** statement can contain function calls.

(iii) A **switch** statement can always be replaced by a series of **if … else** statement.

(iv) The predicate ! ((*x* > = 10) || (y == 50)) is equivalent to (*x* < 10) && (y! = 50).

(v) **while** loops can be used to replace **for** loops without any change in the body of the loop.

OR

Fill in the blanks with appropriate words : **5**

(i) The ___________ specification is used to read or write a short integer. The conversion specifier ___________ is used to print integers in hexadecimal form.

(ii) The ___________ statement when executed in a **switch** statement causes immediate exit from the structure.

(iii) The expression !(*x* ! = y) can be replace by the expression _________.

(iv) The ternary conditional operator ? : could be easily coded using ___________ statement.

(v) In the **do … while** statement if the test expression is evaluated *n* times then the body will be executed by ___________ times.

3. (a) Explain with suitable example : User defined functions in **C** programming language with multiple arguments and multiple return values. **7**

OR

What is mean by a recursion ? Explain by suitable example. **7**

(b) Accepts n integers in an array. Accept an integer and check whether it is present in the array. If it is, display its position and frequency. **6**

OR

Write an algorithm and a program to find the factorial of a given positive integer *n* using a recursive approach.

(c) State whether the following statements are true or false : **5**

(i) The type of all the elements in an array not necessarily be distinct.

(ii) When an array is declared, **C** automatically initializes its elements to be zero.

(iii) If the number of elements to be stored is not known in advance, there may be memory wastage if an array of large size is specified.

(iv) It is possible to increase or decrease the array size during runtime.

(v) The declaration int $x[2] = \{4, 5, 3\}$; is illegal.

OR

Fill in the blanks with appropriate words. **5**

(i) An array that uses more than two subscripts is referred to as _________ array.

(ii) An array can be initialized either at compilation time or at _________.

(iii) A variable declared inside a function is called ___________.

(iv) By default ___________ is the return type of an user defined function.

(v) A function that calls itself is called ___________function.

4. (a) Explain linear stack. Explain queue. What are the differences between stack and queue ? What are the possible operations on a stack and on a queue ? **6**

OR

What is a pointer ? What is mean by pointer to a pointer ? Explain with a suitable example.

(b) Write an algorithm and a program to find the binomial coefficient $\binom{n}{r}$ for a given non-negative integer r and a given positive integer n with $0 \leq r \leq n$. Use an **user defined recursive function** to evaluate the factorial of k. **5**

OR

Write an algorithm and a program to find the greatest common divisor and the least common multiple of two positive integers m and n using a recursive approach.

(c) State whether the following statements are true or false with proper justification. **5**

(i) Pointer constants are the memory address of memory locations.

(ii) Value of a local variable in a function can be changed by another program.

(iii) Pointers to pointers is a term used to describe pointers whose contents are the address of another pointer.

(iv) The keyword **# define** must be written starting from the first column.

(v) Like other statements, a processor directives must end with a semicolon.

OR

Fill in the blanks with appropriate words. **5**

(i) A pointer variable contains as its value the ________ of another variable.

(ii) The only integer that can be assigned to a pointer variable is ________.

(iii) Function __________ is used to dynamically allocate memory to arrays.

(iv) A __________ is an ordered collection of data in which each element contains the location of the next element.

(v) Data structure which contains a member field that points to the same structure type are called __________ structures.