Seat No. : _______________

XC-119

T.Y. B.Sc.

March-2013

Mathematics

Paper – IX

(Graph Theory & Operations Research)

Time : 3 Hours] **[Max. Marks : 105**

Instructions : (1) There are **5** questions. **All** questions are compulsory.

(2) Notations are usual everywhere.

(3) Figures to the right indicate full marks for the question.

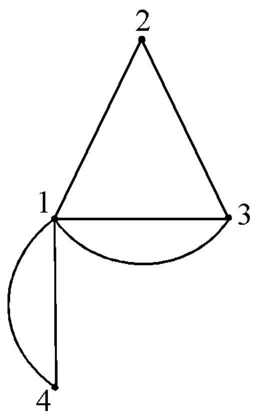
1. (a) Attempt any **three** : **18**

(i) State and prove the Handshaking Lemma. Deduce that the number of vertices of odd degree in a graph must be even.

(ii) Define k-cube graph Q_k. Prove that $\left|V(Q_k)\right| = 2^k$ and $\left|E(Q_k)\right| = k2^{k-1}$.

(iii) If G is a simple graph with at least two vertices, prove that G must contain two or more vertices of the same degree.

(iv) Obtain the adjacency matrix of the following graph G. Also draw the graph whose adjacency matrix is A given as following :



$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

(b) Check the validity of the statements : **3**

(i) There exists a graph having degree sequence (0, 1, 2, 3, 4, 5).

(ii) There exists a 3-regular graph having 7-vertices.

(iii) Every complete graph K_n, (n > 1) is not a bipartite graph.

2. (a) Attempt any **three** : **18**

(i) If G is a simple graph with n vertices, k components, and m edges, prove that $m \leq \frac{(n-k)(n-k+1)}{2}$.

(ii) Prove that a simple graph and its complement cannot both be disconnected.

(iii) If G is simple graph with $n(\geq 3)$ vertices, and if $\deg(v) + \deg(w) \geq n$ for each pair of non-adjacent vertices v and w then prove that G is Hamiltonian.

(iv) If T is a graph with n vertices. Then prove that if T is a tree than it T contains no cycles, and has n –1 edges.

(v) Define the girth of a graph. Obtain the girth of each of the following graphs : $K_{5,7}$, W_8, C_8, K_5, Petersen graph.

(b) Answer in short : **3**

(i) Is k-cube graph Q_3 Eulerian ?

(ii) Check the validity of the statement : "Any simple graph with 10 vertices and 37 edges is connected."

(iii) With usual notation of Cayley's theorem, find the labeled trees corresponding to the sequences (1, 2, 3).

3. (a) Attempt any **three** : **18**

(i) Prove that $K_{3,3}$ is non-planar.

(ii) If G is a connected simple planar graph with $n(\geq 3)$ vertices and m edges, then prove that $m \leq 3n - 6$. Also deduce that if this graph has no triangles then $m \leq 2n - 4$.

(iii) Define self dual graph. If G is a self-dual with n vertices and e edges, prove that $2n = e + 2$.

(iv) If G is a simple graph with largest vertex degree Δ then prove that G is $(\Delta + 1)$-colourable.

(v) Prove that every simple planar graph contains a vertex of degree at most 5.

(b) Answer in short : **3**

(i) State Brook's theorem.

(ii) Check the validity of the statement : "$\chi(P_{2013}) = \chi(C_{2013})$".

(iii) Find the thickness of $K_{3,3}$.

4. (a) Define a convex set. Show that intersection of two convex subsets of R^n is a convex set. Is the union of two convex subsets of R^n, a convex set ? **6**

(b) Attempt any **two** : **12**

(i) A company produces three products P_1, P_2 and P_3. These products three types of ores (raw material) M_1, M_2 and M_3. The maximum quantities of the ores M_1, M_2 and M_3 available are 22 tonnes, 14 tonnes and 14 tonnes respectively. For one tonne of each of these products, the ore requirements are as follows :

	$\mathbf{P_1}$	$\mathbf{P_2}$	$\mathbf{P_3}$
$\mathbf{O_1}$	3	–	3
$\mathbf{O_2}$	1	2	3
$\mathbf{O_3}$	3	2	3
Profit per tonne (₹ in thousand)	**1**	**4**	**5**

The company makes a profit of ₹ 1,000, 4,000 and 5,000 on each tonne of the products P_1, P_2 and P_3 respectively. How many tonnes of the products should the company produce in order to maximize its profits ? Formulate the problem and solve by simplex method.

(ii) Solve the following L.P.P. by two-phase simplex method OR Big-M method :

Minimize $Z = 3x_1 + 4x_2 + 6x_3$

Subject to $3x_1 + 5x_2 + 4x_3 \geq 40$

$6x_1 + x_2 + 2x_3 \geq 50$

$2x_1 + 4x_2 + 3x_3 \geq 60$

$x_1, x_2, x_3 \geq 0$

(iii) Solve the following L.P.P. by applying the principle of duality :

Minimize $Z = 2x_1 + 2x_2$

Subject to $2x_1 + 4x_2 \geq 1$

$x_1 + 2x_2 \geq 1$

$2x_1 + x_2 \geq 1$

$x_1, x_2 \geq 0$

(c) Answer in short : **3**

(i) Check validity of the statement : S_1 and S_2 are convex subsets of R^2 then $S_1 - S_2$ a convex subset of R^2.

(ii) Illustrate a LPP of two variables having no solution.

(iii) Illustrate a LPP of two variables having infinitely many solutions.

5. (a) Attempt any **three** : **18**

(i) What is an assignment problem ? Write the mathematical formulation of an assignment problem. Explain the Hungarian method to solve an assignment problem.

(ii) Prove that every transportation problem has a triangular basis.

(iii) Find the optimum solution of the following Transportation Problem :

Origin ↓	**D_1**	**D_2**	**D_3**	**Supply ↓**
O_1	8	5	6	**120**
O_2	15	10	12	**80**
O_3	3	9	10	**80**
Demand →	**150**	**80**	**50**	**280 Total**

(iv) A marketing manager has five salesman and five sales districts. Considering the capabilities of the salesmen and the nature of districts, the marketing manager estimates that the sales per month (in hundred rupees) for each salesman in each district would be as follows :

	D_1	**D_2**	**D_3**	**D_4**	**D_5**
S_1	32	38	40	28	40
S_2	40	24	28	21	36
S_3	41	27	33	30	37
S_4	22	38	41	36	36
S_5	29	33	40	35	39

Find the assignment of salesmen to districts that will result in maximum sales.

(b) Check the validity of the following statements : **3**

(i) Every Transportation problem is an Assignment problem.

(ii) NWCM is used to solve a Transportation problem.

(iii) Every assignment problem has multiple solutions.