Seat No. : _______________

XU-129

April-2013

Five Year M.Sc. (CA & IT) Integrated (K.S.)
F.Y. M.Sc. (Sem.-II)
Matrix Algebra & Graph Theory

Time : 3 Hours] **[Max. Marks : 100**

1. Answer the following : **20**

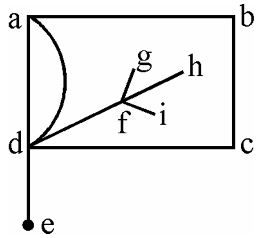
(1) Difference between Bridge and Cut vertex.

(2) Explain Jordan curve with example.

(3) K5 the complete graph is planner or not identify.

(4) Find number of edges of a 4 Regular graph with 6 vertices.

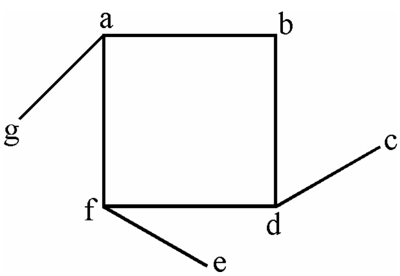
(5) Define following graph G for Euler's formula.



(6) Draw a graph represented by the given adjacency matrix.

$$\begin{bmatrix} 0 & 3 & 0 & 2 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 1 & 2 \\ 2 & 1 & 2 & 0 \end{bmatrix}$$

(7) Find square of the Graph.



(8) What is sub space ?

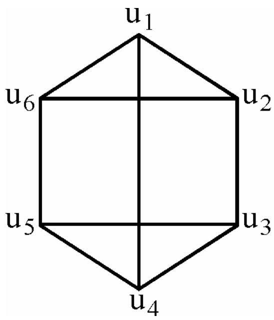
(9) Explain Diagonal matrix.

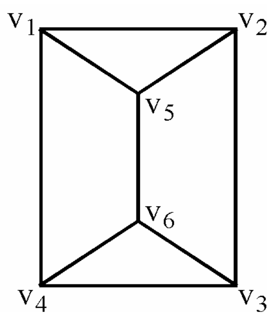
(10) $A = \begin{bmatrix} 1 & 4 & 3 \\ -1 & 9 & 11 \\ 3 & 0 & 5 \end{bmatrix}$

Find cofactor C_{23}.

2. (a) Attempt any three : **12**

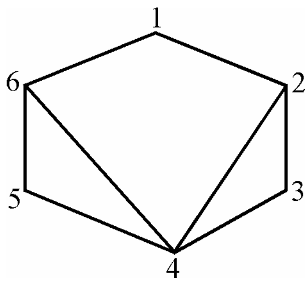
(1) What is isomorphic ? Verify if the given graphs are isomorphic or not ?





(2) State Konigsberg Bridge problem.

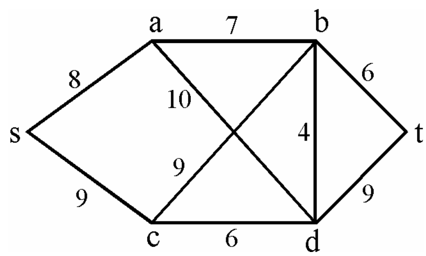
(3) Find closure of graph.



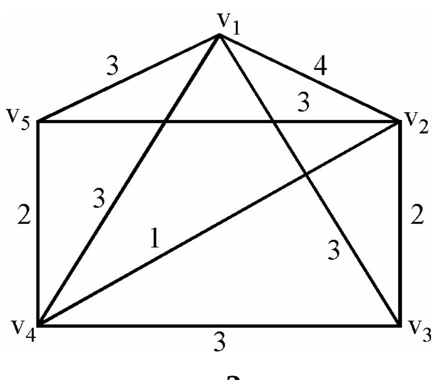
(4) Let G be an a cyclic graph with n vertices and k connected components then show G has n-k edges.

(b) Attempt any **one** : **8**

(1) Apply Dijkaspa's Algorithm for finding the shortest path from 's' to 't'.



(2) What are the main difference between Prim's Algorithm and Krushkal's Algorithm ? Find the minimal spanning tree using Prim's Algorithm and Krushkal's Algorithm of the following graph :



3. (a) Answer the following :

(1) How many edges does a null binary tree with 1000 internal vertices have ? **2**

(2) How many leaves does a null 3-ary tree with 100 vertices have ? **2**

(3) Explain Binary tree with example. **2**

(4) Prove that : **4**

An undirected graph is a tree if and only if there is a unique simple path between any two of its vertices.

(b) Attempt any **two** : **10**

(1) Find the value of a, b, c, *x*, y, z from the following matrix :

$$\begin{bmatrix} a+1 & b+2 & 3+z \\ -5 & c-7 & x \\ x+6 & y+4 & 1 \end{bmatrix} = \begin{bmatrix} 2a+5 & 7 & 2z-5 \\ -5 & 0 & 0 \\ 6 & 5 & 1 \end{bmatrix}$$

(2) If $A = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 2 \\ 0 & 1 \end{bmatrix}$ then prove that

$(A \cdot B)^T = B^T \cdot A^T$

(3) Find solution of given system by matrix inverse method $[X = A^{-1}B]$

$2x_1 - x_2 + 2x_3 = 2$

$x_1 + 10x_2 - 3x_3 = 5$

$x_1 - x_2 - x_3 = 3$

4. Answer the following : **20**

(1) For a given matrix A find A^2, A^{-1}, A^{-2} using Caley Hamilton theorem.

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

(2) Find eigen value eigen vector of the following matrix :

$$A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$$

(3) Using Gauss Jordan method find value of *x*, y, z

$x - 2y + 3z = 4$

$2x - y + 3z = 5$

$-x - y + 2z = 3$

OR

Using Gauss Elimination method find value of x, y, z

$x - 2y + 3z = 4$

$2x - y + 3z = 5$

$-x - y + 2z = 3$

(4) Define Rank of a matrix. Find the Rank of the matrix.

$$A = \begin{bmatrix} 3 & -1 & -2 \\ -6 & 2 & 4 \\ -3 & 1 & 2 \end{bmatrix}$$

5. (a) Attempt any **five** : **15**

(1) Prove that the 3 vector are linearly dependent or not

$x_1 = (1, 1, 1)$, $x_2 = (1, 1, 0)$, $x_3 = (1, 0, 1)$

(2) Find the matrix for the linear transformation $T : R^3 \to R^2$ is defined by $T(x, y, z) = (x + 2y, 3x - y)$

Standard basic

$F_1 = (1, 0, 1)$, $F_2 = (0, 1, 1)$, $F_3 = (1, 1, 0)$

(3) Answer the following :

(a) Given u = (3, 4) find the length of the vector.

(b) Prove that $(-1) \cdot u = (-u)$

(c) Given u = (3, 2) and v = (4, 5) find u.v.

(4) Expand the following term using Binomial theorem :

$(69)^5$

(5) Find the probability that in random arrangement of the letters of the word 'ASSASSINATION'.

(6) Explain Pigeonhole Principle.

(b) Answer the following : **5**

(1) In a class of 15 student 10 are boys and 5 are girls 2 students are selected at random from the class :

(i) find the probability that there is at least one girl in the selection of two students.

(ii) find the probability that there is a boy and girl given condition that there is at least one girl in the selection of two students.